The Experimental Determination of Mental Discipline in School Studies

(DESCRIPTIVE GEOMETRY AND MENTAL DISCIPLINE)

THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN EDUCATION IN THE GRADUATE SCHOOL OF THE UNIVERSITY OF ILLINOIS

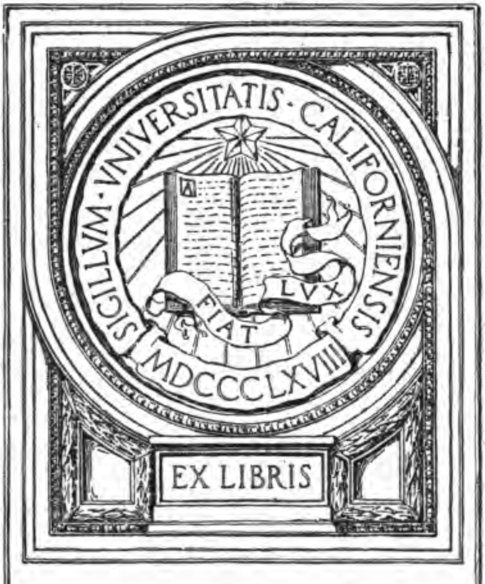
1915

BY

HAROLD ORDWAY RUGG

B.S. Dartmouth College, 1908
C.E. Dartmouth College, 1909

Published by
WARWICK & YORK, Inc.
Baltimore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EDITOR'S PREFACE.

The problem of mental discipline, of determining under what conditions, by what methods and to what extent training received in a given line of mental activity spreads to other lines of mental activity, is acknowledged to be the central problem of educational psychology. The answer that experimental investigation affords to this problem is acknowledged to possess a most significant bearing upon several cardinal problems in education. Thus, for instance, the arrangement of the curriculum, particularly in the high school, the designation of prescribed courses in both high school and college, the selection of methods of instruction in various subjects—all these things depend in part upon what we know about the nature of mental discipline.

Dr. Rugg's monograph claims attention for two reasons especially: (1) it presents in a compact semi-tabular form a valuable and comprehensive summary of all the experimental work that has been done upon formal discipline to date; (2) it presents the results of the author's own investigation, which is conspicuous because it deals with a large number of subjects (students in the University of Illinois), and because it measures the effect upon mental efficiency produced by a course of instruction (descriptive geometry) carried on under regular classroom conditions. The demonstration of a certain degree of transfer of training is of real importance both in educational theory and practice.

G. M. W.

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INTRODUCTION.

Summary of the Aims, Methods, and Results of this Monograph.

The aim of Chapter I is to present in complete, tabular and statistical form the essential features of all the published experiments in formal discipline. To carry out this purpose Plates I, II, and III give the experimental material in detail organized under the following heads: the investigator, date of reporting the study, where studied, reference, the mental abilities under investigation, the subjects (adult or children), number and psychological training of each, description of the test and training series, introspective material, statistical results, conclusions, interpretation, and further interpretive comment. Tables I to V summarize in definite form the typical status of experimentation under each of these items. The conclusions of Chapter I thus set forth the attitude of the present-day psychology on the question of the "transfer of training."

The remainder of the monograph, Chapters II-XI, presents an experimental investigation of the effect of a semester's study of descriptive geometry upon specific abilities in the mental manipulation of spatial elements, (*a*) of a strictly geometrical character; (*b*) of a quasi-geometrical character; (*c*) of a non-geometrical character.

The *general procedure* was to study "transfer" by means of a detailed determination, through classroom experiment, of the spread of training obtained in a specific school course of study.

Four hundred and thirteen college students in the University of Illinois, ranging in age from 17 to 26 years, served as *subjects*. Three hundred and twenty-six freshmen engineering students acted as the *training group,* 87 juniors and seniors of the college of liberal arts and sciences made up the *control group*. Both the training and the control group took the *test series* in February and June. Only the training group took the *training course*, which was the regular University of Illinois course of study offered to engineers in *descriptive geometry*. In order to be able to *isolate the cause of any residual gains* that might possibly be found favoring the training group, this group was further subdivided into seven *sub-sections,* each of which was composed of those students *pursuing like courses of study*. As descriptive geometry was the only course pursued by all the subjects, statistical study could be made of the relative training value of the other courses pursued by the subjects.

This *training course* was a semester's study of descriptive geometry, a prescribed freshman course in the various engineering curriculums. The course is primarily a course in *the solution of spatial problems, involving visual concepts*. It is taught at the University of Illinois primarily as a disciplinary course, as a "pure science," in opposition to the commonly accepted point of view that it is an applied science.

The *test series* consisted of six written class tests for ability in the mental manipulation of spatial elements; Numbers 1, 2, and 6 involved non-geometrical elements, Number 3 involved quasi-geometrical elements, and Tests 4 and 5 involved strictly geometrical elements. Preliminary "laboratory" tests were given during the period September 1912 to January 1913 to 41 subjects, and upon this basis the final tests used in this investigation were

constructed. The final tests were given in February and in June, 1913 and 1914. They were given to the subjects grouped in the regular teaching sections of the department of general engineering drawing, with careful general and specific directions and with classroom conditions as nearly normal as possible.

The *test sheets* were all *scored by the author* and the original tabulations were so made up as to show the absolute amount of work done correctly by each of the subjects for both the February and June test-series. From these original tables other summary tabulations were made showing: (1) *for each subject* the per cent efficiency (February and June), the gain in efficiency, the number and kind of tests in which gains were made; (2) *for the entire training and control groups* the median number of problems solved and the median gains made, the number of subjects making a certain number of "Attempts" and the per cent of this number obtaining a certain number of "Rights", the number of subjects gaining in each of the tests in both "Attempts" and "Rights"; (3) for the *training group* (to isolate the cause of residual gain) the number of subjects gaining in each test and the amount of gain of each subject in each test when the subjects are grouped in sub-sections; (4) for *the training group* the correlation of abilities involved in test and training series, February and June; (5) for the *training group* the effect of training on different grades of scholastic ability, showing the relative number of gainers among high-grade students and low-grade students when grouped according to scholastic ability in (*a*) mathematics and (*b*) English and modern languages; (6) for the *training group* the effect of training at different levels of maturity; (7) *for the training group* the correlation of scholastic ability in various college studies.

Results of the investigation. 1. Statistics of *initial efficiency* of the two groups reveal the *training group* as the *superior* in all tests. Thus a residual gain in favor of the training group cannot be ascribed to a larger possible range of progress because of a lower initial efficiency.

2. *Group efficiency.* In all tests the median gain in attainment of the entire training group exceeds that of the entire control group in both "Attempts" and "Rights." The residual gains in favor of the training group are progressively greater as one proceeds from the non-geometrical tests (Numbers 1, 2, and 6) in which the average residual gain is 7 per cent to the quasi-geometrical test (Number 3) in which the residual gain is 20 per cent, and to the strictly geometrical tests (Numbers 4 and 5) in which the residual gain is 31 per cent.

3. *Individual efficiency.* (*a*) *Number of subjects gaining.* "Attempts:" in all tests, approximately 50 per cent more of the training subjects gain than of the control subjects. "Rights:" considerably over half again as large a proportion of the training group gain as of the control group. (*b*) *Amount of gain.* In both "Attempts" and "Rights" the training group gains approximately 20 per cent more than the control group and makes a progressively greater gain as one proceeds from the tests with non-geometrical elements to the tests with quasi- and with strictly geometrical elements. (*c*) *Number of tests in which gains were made.* "Attempts:" 67.8 per cent of the training group and 42.5 per cent of the control group gain in 60 per cent or more of the tests taken. "Rights:" 72.7 per cent of the training groups and 31 per cent of the control group gain in 60 per cent or more of the tests taken. The average per cent of tests for which gains were made, is for the training group: "Attempts" 63 per cent; "Rights" 64.5 per cent; for the control group: "Attempts"

49 per cent; "Rights" 40 per cent. In general, the training group gains in a distinctly larger proportion of the tests taken than does the control group.

4. *Correlations of efficiency in the various tests (February and June). Training group:* The correlations of ability in various tests are all positive, ranging around +.30. For Tests 1 to 5, inclusive, 11 out of 12 of the June correlations are noticeably higher than the corresponding correlations for the February results. Many have doubled. *Control group:* The June correlations are practically identical with those obtained from the February tests. The correlations confirm the results found above as to the effectiveness of transfer, and emphasize the complexity of interaction of such seemingly simple mental abilities as the manipulation of spatial elements.

5. *Effect of the training upon students of different grades of scholastic ability.* When the students are grouped on a basis of scholastic ability in disciplinary or "problem" courses (such as mathematics courses), the semester's training received in descriptive geometry is found to 'spread' more effectively with the high-grade than with the low-grade students. When the students are grouped on a basis of scholastic ability in English and the modern languages, the training is found to be equally effective with all grades of scholastic ability This difference suggests the possibility that the "method of transfer" is to be defined largely in terms of conceptualizing and organizing abilities.

6. *Effect of training on subjects of various ages.* Within the limits represented by this study, the age of the subject does not seem to be a factor in determining the effect of training upon his ability in the mental manipulation of spatial elements.

7. *Correlation of scholastic ability in various college*

studies. The scholastic abilities of the subjects, as measured by their college marks, are positively correlated to a rather marked degree; the values of *r,* the coefficient of correlation, range from +.44 to +.70.

8. *The agencies of transfer.* It is concluded from the results of this study that the agencies of transfer are the following: (1) Many specific adjustments of reactions to familiar "cues" of visualization have tied together training and test series and have undoubtedly acted as bonds or agencies through which more efficient response is made to a given situation. (2) An important phase of the successful response to a new situation is the building up of attitudes of orientation in the general visual field. (3) Practice in extending the "range of attention" has given increased facility in holding and manipulating a number of visual elements at the same time. (4) The study of the effect of training on various grades of scholastic ability points to the possible effectiveness of conceptualizing abilities in developing methods of analysis or attack. The introspective material confirms the view that these conceptualizing abilities play a part in determining the method of transfer.

9. *The disciplinary outcomes of school studies.* The possibility of one disciplinary outcome of a specific school subject, *i. e.,* the ability in the mental manipulation of spatial elements, has been established in this investigation. The experimenter believes that, in general, disciplinary outcomes of school studies will be found in the above-listed agencies of transfer, *i. e.,* the development of concepts of method in analyzing 'problem' situations and organizing methods of procedure, the habitualizing of reaction to specific cues, the development of attitudes of orientation and familiarity with the type of situation to be met, and the extension of the range of attention.

CHAPTER I.

The Essential Features of the Experimental Literature of Mental Discipline.

When William James, in his pioneer study of the "transfer of training" in the memorizing of verse (Plate I, 1) suggested that the psychologist and the educationist apply the experimental method to the solution of problems of formal discipline, a new line of attack was marked out. The 'arm-chair' philosophy, which for twenty centuries had settled *a priori* all questions of the disciplinary values of studies, now gave way to an experimental laboratory method. The literature developed rapidly; the philosophical educationists still continued to add to its volume through their "symposiums" and through their critiques of the various empirical attacks on the problem. Thus, after a quarter-century of empirical discussion, students of mental discipline are beginning to reach certain common grounds of agreement.

To the careful, trained student of the literature, wading laboriously through study after study, the outcome may be a fairly definite classification of experiments, methods, subjects, tests, statistical results, conclusions, and interpretations. It may be that such a specialist can carry away from months of intensive study of transfer a clear composite picture of the possibilities and methods of improving related and unrelated mental abilities. But to the beginner in the study of education the problem is daily becoming more and more complex. A rapidly growing experimental literature calls for new methods of presentation, classification, and interpretation. In the belief

that there is a real need for a thoroughly comprehensive and yet definitely organized presentation of the experimental literature of formal discipline, the author aims in this chapter to present a graphic summary of all the experimental work that has been published in this field. The aim is to *present the essential facts in a form that will clearly reveal their implications,* and thus to supply the student with a guide to his study of the problem, through *a graphic and more logical organization of the facts* than is to be found in the many detailed summaries now in print. The data will be presented in tabular and statistical form and the interpretations will be brief.

To carry out this plan, the essential features of the published literature of formal discipline are presented on Plates I, II, and III, and in still further summarized form in Tables I to V. It is believed that we have now reached the place where it is justifiable to *apply the statistical method to the study of the literature itself.* Indeed, probably the most efficient study of the experimental results will come from a preliminary quantitative classification of the work that has already been done.

On Plates I to III the experimental data have been organized under the following heads: the mental abilities studied, the method of study, the number of subjects—total, 'training,' and 'control'—ages and psychological training of the subjects, description of the test and training series, statistical results, introspective data for the study of the agencies of transfer, the experimenter's interpretations, conclusions, and further interpretative comment. The writer is alone responsible for the method of tabular presentation, the reading and organization of

TABLE I.

DATA CONCERNING THE DATE OF PUBLICATION OF TRANSFER STUDIES AND THE GENERAL PROCEDURE FOLLOWED.

Investigator	General Method		Date of Publication				
	Lab.	School	1890-1895	1896-1900	1901-1905	1906-1910	1911-1916
Memory Studies:							
James	L	..	x	..	..	..	..
Peterson	L	..	..	..	..	..	x
Ebert and Meumann	L	..	..	..	x	..	..
Dearborn, '09	L	..	..	..	..	x	..
Fracker	L	..	..	..	..	x	..
Winch, '08	..	S	..	..	..	x	..
Winch, '10	..	S	..	..	..	x	..
Dearborn, '10	L	..	..	..	..	x	..
Sleight	L	S	..	..	..	..	x
Sensory and Perceptual Experiments:							
Thorndike and Woodworth	L	..	..	..	x	..	..
Judd	L	..	..	..	x	..	..
Coover and Angell	L	..	..	..	..	x	..
Bennett	L	..	..	..	..	x	..
Scholkow and Judd	L	S	..	..	..	x	..
Kline	L	..	..	..	..	x	x
Wallin	L	..	..	..	..	x	..
Whipple	L	..	..	..	..	x	..
Foster	L	..	..	..	..	..	x
Dallenbach	..	S	..	..	..	..	x
Gilbert and Fracker	L	..	..	x	..	..	..
Jastrow	L	..	..	x	..	..	..
Bair	L	..	..	..	x	..	..
Leuba and Hyde	L	..	..	..	x	..	..
Ruger	L	..	..	..	..	x	..
School Activities:							
Bagley (Squire)	..	S	..	..	x	x	..
Ruediger	..	S	..	..	..	x	..
Bennett	L	..	..	..	..	x	x
Briggs	..	S	..	..	..	..	x
Wallin	..	S	..	..	..	..	x
Rugg	..	S	..	..	..	..	x
	22	10	1	2	6	15	9

material contained in these studies, and the transcription of their essential features on the large plates.*

Following the more obvious methods of organization, the studies are grouped under three main heads: (*a*) studies dealing with memory abilities; (*b*) studies dealing with sensory, perceptual and associative motor habits; (*c*) studies dealing with the specific processes involved in certain school subjects.

We proceed at once to an analysis of the experimental literature.

Prior to any experimental study of mental abilities and the transfer of improvement in their training, psychologists gave us a few physiological studies of so-called 'transfer' in the case of bilaterally symmetrical organs. These studies have sometimes been quoted as offering evidence bearing on the problem of mental discipline. The writer believes, with Thorndike, that the argument by analogy for the transference of training of mental abilities, based on these physiological studies, is an improper one, and does not contribute to the solution of our problem. As Thorndike says: "The phenomena of cross-education are an interesting chapter in experimental psychology, but are not fair samples of the general facts we seek."

From Table I it is clear that only within the past decade has any attempt been made to study mental discipline by

*It is recognized that in the organization and transcription of so complex a mass of data the chance of occasional error is large. It may be said, however, that the collection of material and the organizing and lettering of the plates have been carefully checked several times. It is believed that all errors of consequence have been eliminated, and that the student may safely use the presentation as a definite and comprehensive guide for the study of the experimental literature of transfer. The work has been done with the recognition of the fact that the investigations permit only partially the definite type of classification represented by these plates and tables. The presentation is to be regarded only as a device to aid the student in formulating his information.

TABLE II.

DATA CONCERNING SUBJECTS OF THE TRANSFER EXPERIMENTS.

	Number of Subjects						Ages or Grade in School		Extent of Psychological Training				Kind of Intro-spective Material Secured
	Total		Training		Control				Teacher of Psych.	Grad. Stud. Psych.	Some Psych. Training	No Psych. Training	
INVESTIGATOR	Adult	Ch'n	Adult	Ch'n	Adult	Ch'n	Adults	Ch'n					
MEMORY													
James	5		5				Grad. St. & Teachers		x	x			
Petersen	9		2		7		Teachers & Norm. Stud's		x	x			
Ebert and Meumann	6		6										
Dearborn, '09	No Data												
Fracker	12		8		4				x	x	x		Good
Winch, '08		34 34 54		17 17 27		17 17 27		Standard III, IV, & V				x	
Winch, '10		40 46 34		20 23 17		20 23 17		Standard III, IV, & V				x	
Dearborn, '10	12		12				Coll. Stud's				x		
Sleight	18	84					18-19 Yrs.	12 Y, 8 M			x	x	Slight Value
SENSORY, PERCEPTUAL and MOTOR EXPTS.													
Thorndike and Woodworth	6 5 4		6 5 4				Coll. Stud's				x		
Judd	2		2				Teachers Psych.		x				
Coover and Angell	6 4		4 1		2 3		Coll. Stud's				x		Good
Bennett	2 16 2		2 16 2				Teachers & Coll. Stud's		2=x			x	
Scholkow and Judd	1 2	2 Gr'ps Boys	1 2	1 Group		1 Group						x	
Kline	18 22		9 ?		9 ?		Coll. Stud's					x	A Few Data
Wallin	2		2				Teachers		x				
Whipple	6 3		6 3				Teachers & Coll. Stud's		x		x		Good
Foster	3		3				Teachers & Grad. Stud's		x	x			Good
Gilbert and Fracker	3		3				Teachers Psych.		x				
Jastrow	?		2		Miscel. Group		Prof. Magicians Coll. Stud's				x	x	
Bair	4 3		4 3				Coll. Stud's				x		
Leuba and Hyde	42		26 16				Coll. Stud's				x	x	
Ruger	27		27				Coll. Stud's Mechanics	4 Grammar School Boys	7=x		15=x	15=x	
Dallenbach		44		29		15		9 Yrs. Old				x	
SCHOOL ACTIVITIES													
Bagley-Squire		2 Groups		1 Group		1 Group		3d Grade				x	
Ruediger		39						7th Grade				x	
Bennett	27 to 145		27 to 145				Coll. Stud's					?	
Briggs		47 295		23 130		24 61		6th, 7th, 8th Grades				x	
Wallin		1022		1022				4th to 8th Grade				x	
Rugg	500		413		87		17 to 26 Yrs.				87 x	413 x	Some "Good"

reference to experimentation with schoolroom activities. In fact, ten years elapsed following James' indication of non-transference before a concerted attempt to work out the problem in the laboratory was reported. This laboratory phase of the movement has engaged the attention of most of the investigators since 1900. The work has been done principally in connection with so-called 'peripheral' functions, more or less isolated perceptual and sensory processes, and, in general, quite apart from the concrete and complex situations of actual mental life. More recent investigators are recognizing the inadequacy of the former attack and are concerning themselves with the classroom problem. It would seem that the application of the results of transfer studies to the advancement of school practice can be of immediate value only when the studies themselves have to do with school practice, are conducted under normal school conditions and deal with 'real' mental situations such as are met with in our school practice and in our daily mental life.

Table II summarizes the data concerning the subjects used in the transfer studies. They are characterized by three important features: (1) the subjects have nearly always been adults; (2) they have been few in number; (3) they have been a highly selected class, usually teachers of psychology or students with considerable psychological training. In only nine experiments have children been tested and studied. From the point of view of educational practice this is to be regretted and it is to be hoped that future experimenters will take notice of this need. Furthermore, the number of subjects in most investigations has been so small as to render questionable the generalizations that have been made in interpreting the results of the experimentation and in drawing inferences for school practice. In 12 of the 30 experiments, the number of subjects was six or less. The few class-

room experiments have been characterized by a large number of subjects, usually 20 to 50.

The use of control groups for the determination of the effect of (*a*) practice in the test series, (*b*) increased maturity, and (*c*) familiarity with experimental conditions is rather recent in experimentation. The experiments in memory by Fracker, in motor response by Schol-kow and Judd, and the studies of Bagley and Ruediger in connection with practical school activities were the first to take account of these factors. As is indicated on Plates I to III, the conclusions of certain of the experiments are invalidated by the neglect of the experimenter properly to control his experiment. Particularly is this true of James' study and of the extensive memory investigation of Ebert and Meumann. The more extensive studies, and particularly the classroom studies, are little open to criticism on this score.

Psychological training of the subjects. Nothing shows more clearly the selected nature of the subjects of the transfer investigations than the status of their psychological training. The laboratory experiments have nearly all been performed on trained psychologists, graduate students in psychology, or persons with some psychological training. It might seem that their arrangement would afford valuable introspective analyses of the mechanism of transfer. That there were secured few actual contributions to the discussion of the agencies of transfer is shown by the table, which indicates that of 19 investigators using subjects trained in psychology, only five were able to secure careful introspections, namely, Fracker, Coover and Angell, Whipple, Foster, and Rugg. Naturally, any attempt to determine the mechanism of transfer without the specific aid of the subject himself is likely to contribute but scanty suggestions for the advancement of school practice. On the other hand, the classroom ex-

TABLE III A.

TEST AND TRAINING SERIES USED IN STUDIES DEALING WITH MEMORY FUNCTIONS.

TRAINING SERIES ↓ / TEST SERIES →	Memory of poetry.	Memory of series of letters, words, numbers, visual forms, etc.	Memory of prose paragraphs.	Memory of the order of intensity of colors or sounds.	Motor movements.	Memory of spatial relations, dates, map location, nonsense syllables, dictation, etc.
Memory of poetry.	James Peterson Winch, '08 Winch, '10 Sleight					
Memory of nonsense syllables.	Ebert and Meumann Dearborn, '09	Ebert and Meumann Dearborn, '09	Ebert and Meumann Dearborn, '09			Ebert and Meumann Dearborn, '09
Memory of the *order* of intensity of tones.	Fracker			Fracker	Fracker	
Memory of prose substance.						Sleight
Memory of tables of various kinds.						Sleight
Memory of foreign and English vocabularies.	Dearborn, '10		Dearborn, '10			
Memory of meaningless things.	Winch, '10		Winch, '10			

TABLE III B.

TEST AND TRAINING SERIES USED IN STUDIES DEALING WITH SENSORY, PERCEPTUAL AND ASSOCIATIVE-MOTOR FUNCTIONS

THE TEST SERIES.

THE TRAINING SERIES.	Estimation of weights, lengths, figures, etc.	Estimating the length of line in Müller-Lyer illusion.	Discrimination of brightness stimuli.	Discrimination of differences in pitch.	Accuracy in observation of certain forms.	Ability to hit targets under water.	Accuracy of hand movement.	Cancellation of certain forms.	Substitution of certain letters, digits, etc.	Tachistoscope testing of quickness in perception, apprehension, etc.	Reaction to sound, color or shock stimuli.	Typewriter reaction to certain stimuli.	Learning alphabet—other methods.	Writing of English or German script.	Solution of mechanical puzzles.
Estimation of weights, lengths of lines, size of figures, etc.	Thorndike and Woodworth														
Estimating length of line in Müller-Lyer illusion.		Judd													
Discrimination of sound stimuli.			Coover and Angell												
Estimating lengths by arm movement, accuracy of hand movement.	Bennett						Judd								
Discrimination of color saturation.	Bennett		Bennett	Bennett											
Accuracy of observation of certain shapes, figures, etc.					Judd										
Study of theory of refraction						Judd									
Cancellation of certain forms.								Kline							
Substitution of numbers and symbols.									Kline						
Tachistoscope training in quickness of visual perception: apprehension.										Whipple Foster Dallenbach					
Quickness in reaction to sound stimuli.											Gilbert and Fracker				
Many years' experience in sleight of hand.	Jastrow		Jastrow		Jastrow		Jastrow		Jastrow		Jastrow				
Typewriter reaction to certain stimuli.	Bennett						Bennett	Bennett				Bair			
Learning alphabet by certain method.													Bair		
Practice in sorting cards.												Coover and Angell			
Writing German or English script.														Leuba and Hyde	
Solution of "mechanical" puzzles.															Ruger

TABLE III C.

TEST AND TRAINING SERIES USED IN "SCHOOL ACTIVITIES" STUDIES.

THE TEST SERIES.

THE TRAINING SERIES.	Tested for neatness in written work in other school subjects.	Various perceptual abilities, reaction time, memory, quickness in marking A's, etc.	Ability to see likenesses and differences, to judge, amend and apply definitions, to make and use rules, arithmetic and other reasoning abilities, etc., etc.	Ability in the mental manipulation of spatial elements (non-geometric and geometric types).	Tested in column drill and dictated composition method in spelling.
Training in "neatness" in written work in one school subject.	Bagley-Squire Ruediger				
General training in four years' college course.		Bennett			
Three months' training in formal grammar.			Briggs		
Four months' training in descriptive geometry				Rugg	
Training in column-drill methods of spelling.					Wallis

periments have been upon children, and they have studied in some objective way the efficiency of certain mental processes that are involved in specific school activities. The objective results may lead to a quantitative determination of the effect of training on school abilities. In this, however, the experimenter faces almost insurmountable obstacles in deriving an explanation of the method of transfer.

The test and training series used in various types of studies. What are the salient facts concerning the data that have been under examination in these transfer studies? In the first place, Tables IIIa, IIIb, IIIc show the data to be extremely diverse and unorganizable. That is, almost none of the mental abilities studied has been studied by more than one investigator. We have few instances, in twenty-five years of experimentation, of either co-operative or independent study of the effect of training upon the same mental abilities, and we have yet to find a detailed laboratory or classroom study that has been repeated under precisely the *same conditions* and in the *identical form* of the original study.

This points to an experimental need in our work. Our departments of experimental education and educational psychology would do well to take up this problem of 'checking up' those transfer experiments that seem to offer positive evidence on either side of the question of transfer. The product of a few years of organized class study could thus shed valuable light upon the present unsettled status of many of the issues.

Another pertinent induction that may be made from the tables is that nearly all the abilities studied may be denoted as 'peripheral' functions. They utilize to only a limited extent the higher powers of observation and reasoning, and are largely inapplicable to the complex situations of our actual every-day mental life. This important

TABLE IV.
STATISTICAL RESULTS OF TRANSFER EXPERIMENTS.

Investigator	No transfer	Statistical evidence doubtful	Slight gains indicate some transfer	Clear evidence of considerable transfer	Experimenter claims large amount transfer, unsupported by evidence	Repetition of experiment shows transfer
Memory Studies:						
James	x	..	x	..	..	x
Peterson	..	..	x	..	..	x
Ebert and Meumann	..	..	..	..	x	x
Dearborn, '09	..	..	..	..	..	x
Fracker	..	..	x	x	..	..
Winch, '08	..	..	x	..	..	..
Winch, '10	..	..	x	..	..	..
Dearborn, '10	..	..	x	..	..	..
Sleight	..	..	x	..	..	..
Sensory and Perceptual Experiments:						
Thorndike and Woodworth	..	..	x	..	..	..
Judd	..	..	..	x	..	..
Coover and Angell	..	x	..	..	x	..
Bennett—1 and 3	x	..	..	..	..	..
2 and 4	..	..	x	..	..	..
Scholkow and Judd	..	..	..	x	..	..
Kline	..	..	x	..	..	..
Wallin	..	..	..	x	..	..
Whipple	..	..	x	..	..	..
Foster	..	..	x	..	..	..
Dallenbach	..	..	x	x	..	..
Gilbert and Fracker	..	..	..	..	x	..
Jastrow	..	x	x	..	..	..
Bair	..	..	..	x	..	..
Leuba and Hyde	..	..	..	x	..	..
Ruger	..	..	x	x	..	..
School Activities:						
Bagley—Squire	x	x	..	..	..	x
Ruediger	..	..	x	..	..	..
Bennett	x	..	..	..	..	..
Briggs	x*	..	x	..	..	..
Wallin	..	..	..	x	..	..
Rugg	..	..	..	x	..	..

*Doubtful interpretation of data reported.

TABLE V.

THE AGENCIES OF TRANSFER.

Investigator.	Effect of practice is specific; improvement spreads only through presence of identical elements.							Effect of practice may be generalised. —Through— Ideational factors.		Attention factors.			Attitudinal factors.		
	No discussion of agencies of transfer.	General claim for identical elements.	Identity in content.	Identity in procedure.	Identity in aim.	Specific habits.	Perceptual habituation.	Recognition of identical elements plus ability to use them.	Improvement in methods of learning; technique of learning; grouping tricks; ideals of method of analysis or attack; "conceptualizing" abilities.	General function of habituation (hypothetical).	Methods of distributing attention.	Incr. power concentration; high level attention; extending range.	Attitudes of orientation and adjustment; familiarity with exp't'l conditions.	Confidence of success. Central predisposition to meet new situations.	Interference operates to block transfer.
James	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Peterson	..	..	..	..	..	..	..	..	x	..	..	..	..	..	..
E. and Meuman	..	..	..	..	..	..	..	..	x	x	..	x	..	..	..
Dearborn	..	..	..	..	..	..	..	..	x	..	x	x	..	..	..
Fracker	..	..	..	..	..	..	..	x	x	..	x	..	..	..	..
Winch, '08	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Winch, '10	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Dearborn	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Sleight	..	..	x	x	x	..	..	x	..	..	..	..	..	..	x
T. and Woodworth	..	x	x	..	..	..	..	..	..	..	..	..	..	..	..
Judd	..	..	..	..	..	..	x	..	..	..	..	..	..	..	..
C. and Angell	..	..	..	..	..	..	..	..	x	x	x	..	..	..	..
Bennett	x	..	..	..	..	..	..	..	..	..	..	..	..	..	x
S. and Judd	..	..	..	..	..	..	..	..	x	..	..	..	..	..	x
Kline	..	..	..	..	..	..	..	..	..	..	x	..	..	..	..
Wallin	..	..	..	..	..	..	..	..	..	..	..	..	..	x	..
Whipple	..	..	..	..	..	..	..	..	x	..	..	..	x	..	..
Foster	..	..	..	..	..	x	..	..	x	..	x	..	x	..	..
Dallenbach	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
G. and Fracker	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Jastrow	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Bair	x	..	..	..	..	..	..	..	..	..	..	..	..	..	x
L. and Hyde	..	x	..	..	..	..	..	..	..	..	..	..	..	..	..
Ruger	..	..	..	x	x	..	..	..	x	..	..	x	x	..	..
Bagley—Squire	..	..	..	..	..	..	..	..	x	..	..	..	..	..	..
Ruediger	..	..	..	..	x	..	..	..	x	..	..	..	..	..	..
Bennett	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Briggs	x	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Wallin	..	..	..	x	x	..	..	..	..	..	..	..	..	..	..
Rugg	..	..	..	..	..	x	..	..	x	..	..	x	x	..	..
	10	2	2	3	4	2	1	2	12	2	5	3	4	1	4

consideration will be taken up later in the discussion of the *agencies of transfer*. Thus, in résumé, we see that the abilities studied have been for the most part isolated peripheral functions; they have been studied principally under laboratory conditions, with a few adult subjects, most of whom have had considerable psychological training.

Statistical results of the transfer experiments. Does training transfer? Under conditions of training studied in these thirty investigations we can answer unequivocably: *There is distinct evidence for the so-called transference of training.* The experimental training of the abilities of either adults or school children, in either laboratory or schoolroom, will result in an increased efficiency on the part of the subjects, in *other abilities which are in some way related, to the trained abilities.* In these thirty investigations there are only four in which there is declared to have been found absolutely no evidence of transfer. Moreover, three of these have been partly repeated or 'checked up' statistically, and distinct evidence of transfer found. On the other hand, the experimentation has not led to the acceptance of a belief in that widespread improvement that was expected by the old formal-disciplinists prior to the beginning of the experimental work. The results, so far, place us still in a middle ground. "Transfer" is an accepted experimental fact, but as to the extent to which training transfers and the most favorable conditions for its transfer, specialists are not always agreed. However, the assembled data on the fundamental question of the method or the agencies of transfer (Table V) indicate a tendency of specialists to stress certain similar features in their explanation of their experimental results.

The investigators may be grouped in two schools: (1) those who believe that the effect of training is quite spe-

cific and who oppose the view that transfer can be possible through any form of "generalization"; (2) those who believe that the effect of practice can be generalized. Numerically the latter are much the stronger. Thorndike and Woodworth, Sleight, Leuba and Hyde, and Wallin, claim that improvement can spread only through the presence of identical ability-conditioning factors, such as identity in content or substance, identity in method, identity in ideals or aim, etc. To these may be added Fracker and Sleight, who claim the possibility of transfer through identical elements, provided that the subject consciously recognizes and is able to use the favorable factors as such. This method of explaining transfer is clearly to be classed, however, with the "general-practice-effect" group, among the "ideational" factors. It should be noted that the identical-elements theory of explaining transfer in a way begs the entire question of the method of transfer. It says too much; it is too all-inclusive and might fairly be interpreted as supporting the view of the other school.

Out of the nineteen investigators who contribute to the discussion of the method of transfer (eleven others offer nothing to this most fundamental aspect of the problem), fifteen take the position that transfer is possible through certain factors of generalization. These factors of generalization may be grouped as, (1) ideational factors, *e.g.*, abilities in the organization of methods of procedure, conceptualizing abilities, the development of improved methods of learning, etc.; (2) attention factors, *e.g.*, improved methods of distributing attention, of concentrating, of extending the range of attention, (3) attitudinal factors, *e.g.*, the development of attitudes of (*a*) orientation in general fields of reaction, (*b*) adjustment to experimental conditions, (*c*) confidence in success, etc. Thus the burden of the argument is to the effect that transfer is possible through the generalization of various 'central' func-

tions. (This in itself may explain the slight amount of transfer that has been found, for most of the experimentation has been upon so-called "peripheral" functions—reaction time, speed, and the like, in which motor co-ordinations have been of the simplest sort—those in which 'central' modes of improvement are least effective.)

The experiments indicate that the central improvement consists in devising methods of learning, *e.g.*, tricks and short-cuts for meeting problematic situations. As one writer has put it: "Our instruments do not improve; we only learn to use them better. Those who do not learn to use their instruments . . . from practice, show little or no transfer of improvement through practice." It is pointed out that improvement should be spoken of as 'central' or 'peripheral' rather than as 'general' or 'special' —as a better understanding of how to use the organs of sense rather than as a change in the constitution of these organs. The experiments show that we must distinguish between the ideational possibilities of transferred improvement and the vain hope of the 'spreading' function of rigidly developed sensory, perceptual, and motor adjustments. These latter have to be taken over into new situations unchanged and can operate with increased efficiency only as the conscious utilization of them in combination has been made more effective through experience.

Thus the studies indicate that *the law of learning has to be made a conscious matter of ideation* in order to insure any considerable amount of transferred improvement. The largest improvement seems to come when the subject discovers that certain methods are helpful. In nearly all these investigations we can find constant insistence on securing the aid of ideational devices in order to bring about improvement. Even identical-elements explanations, such as those offered by Thorndike and Woodworth can be regarded as the development of methods of attack-

ing problems through the organizing of methods of procedure. And it seems likely that the cases of no transfer may be accounted for on the ground that the situation had not been raised, or could not be raised, to the plane of conscious effort.

Summary.

In résumé, let us list the fundamental outcomes of the experimental literature of mental discipline.

1. Prior to 1890 no empirical study of the problem of mental discipline had been made, and up to 1900 only three studies were published. Thus, the experimentation in mental discipline is practically a matter of the past sixteen years.

2. The studies have been primarily of 'peripheral' functions, as (*a*) studies dealing with memory abilities, (*b*) studies dealing with sensory, perceptual data or motor-habit-formation, (*c*) studies dealing with specific school activities. These last are few in number and of recent date.

3. The method of study has been principally a laboratory method. Little attempt has been made to study transfer through schoolroom investigation of those 'real' abilities most utilized in the activity of our mental life. Thus, these investigations are subject to criticism on the ground of the purely experimental conditions under which they were conducted, and generalization from them to actual mental situations of life is unsafe.

4. The subjects have generally been adults, selected for their psychological training, and, as a rule, very few in number; to use fewer than six subjects has been typical practice. With the more recent attempts to make schoolroom investigations, school children have been selected as subjects; their number as a rule has been about 20, and the abilities studied have been connected with school activities.

5. But few experimenters have secured definite and detailed introspections to aid them in determining the agencies of transfer.

6. The tests, like the functions studied, may be characterized as largely unreal, unique, and not closely related to the problem-situations of daily mental life. Like the conditions of investigation, they are 'experimental' in character. It should be said that they probably do measure rather well the functions under examination. Of the abilities selected for testing, almost none have been studied by more than one investigator.

7. The statistical results indicate almost unanimously that training does transfer. Within the limits of the investigations, the amount of transfer does not seem very pronounced. Judging from the best of the published investigations, it seems possible that with more extended experiments a more decided amount of spread would be found. With a greater emphasis on the central functions in our selection of abilities for experimentation, we may well expect a larger degree of transferred improvement.

8. By one school of specialists, training has been regarded as specific in effect and transfer has been explained as due to "identical ability-conditioning factors." However, the typical attitude taken today is that practice may be generalized and transferred through such factors as: (1) ideational factors, (2) attention factors, (3) attitudinal factors. Thus, transfer is possible with central functions through the generalization of various ones of these factors. The emphasis here is on making the method of learning a conscious matter, the conscious organization of methods of procedure, the conscious utilization of methods of improvement, better understanding of how to use mental tools, rather than any transferable change (through practice) in the constitution of the organism itself.

CHAPTER II.

Descriptive Geometry and Mental Discipline.*

The Problem, the Method and the Abilities Investigated.

A. The Problem.

> The problem is to investigate the effect of a semester's training in descriptive geometry upon specific abilities in the mental manipulation of spatial elements, (*a*) of a strictly geometrical type; (*b*) of the quasi-geometrical type; (*c*) of a non-geometrical type.

That this problem is one of extreme complexity, we cannot doubt. When we select a regularly organized course of study from one of our higher college or university curriculums as material for experimental psychological analysis, we definitely put behind us the possibility of that exactitude of analysis and that precision in solution that are the goals of laboratory research. No course of study from our higher curriculums could be found which would train one specific mental ability and that alone. All courses exert a training effect upon countless abilities, such as memory, accuracy in observation, accuracy in recording auditory or visual imagery, selective judgment, discrimination of a sensory or perceptual type, etc. Our endeavor, then, must be to select abilities for training whose activity may be well isolated and whose efficiency may be measured at least approximately by mental tests.

Picture for a moment the difficult situation before the investigator who attempts, *e. g.*, to determine what training effect a year's study of high-school algebra has upon

*The writer wishes to acknowledge the kindness of the various members of the Department of General Engineering Drawing, University of Illinois, in permitting the tests to be given in their classes in 1913 and 1914. To Mr. F. M. Porter of this department he is especially indebted for hearty co-operation and many valuable suggestions, and to Dr. W. C. Bagley and Dr. G. M. Whipple of the Department of Education for critical reading of the manuscript and important suggestions as to methods of treating and presenting the material.

certain mental abilities. His subjects are statistically a heterogeneous class, of various ages, of various previous training in mathematics, registered in diverse courses or curriculums, having a variety of daily interests and taking part in a variety of daily activities both in school and out. If he has many subjects, they are necessarily distributed in several sections under different instructors, and thus subjected in their training to different organizations of the material in the training course itself and to diverse methods of teaching. He further faces the difficulty of selecting a mental ability trained by his training course, but which other concurrently taken courses do not also train to an appreciable extent. Again, granted that the desired mental ability has been isolated for work, our investigator must next devise a series of mental tests which will be a real measure of efficiency in the selected ability. This, in itself, is no small task, and may well be the first point of attack by critics of such investigations.

The tests once organized, the mechanism under which they are taken by the subjects must be standardized, classroom conditions kept normal, and all outside (and inside) influences of possible effect upon the results must be eliminated in the conduct of the investigation. In the conduct of the investigation the attitude of the subjects toward the investigator giving the tests must be regarded as an imporant phase of the mechanics of test-giving. A change in the prestige which the investigator has in the eyes of his subjects may exert a powerful influence upon the psychological product obtained. Another important phase of the test-giving is the securing from the subjects themselves of definite introspections on the various tests taken. Nothing is of so much import to the accurate analysis and interpretation of the statistical presentation of the psychological product, and yet nothing in the con-

duct of the investigation is more difficult to secure from students who have not been specifically trained in introspection. Going further, the very scoring and recording of the data—even with the aid of thorough introspections on the part of the subjects themselves—raises constant doubt as to the measurability, by certain bits of mental product (the test results), of the efficiency of the accompanying mental process. Nothing but a continuous and minute oral interpretation by each subject of each of his tests (made and recorded immediately after taking the tests) would enable the judges to score accurately such tests, so that they would indicate the precise efficiency of the mental ability under observation. This, of course, is impossible in an extensive investigation, and to be valid, any such practical school investigation must be extensive.

Finally, his tests scored and recorded, his tables and accompanying illustrative diagrams drawn up, our investigator faces his last and perhaps greatest problem. From statistical generalizations, which his attack on the problem may have enabled him to devise (such as averages, measures of dispersions, per cent efficiency, absolute quantities of work done, absolute gains of various groups, per cent gains of various groups, correlations between efficiency and gain of various groups, etc.), he must now interpret the efficiency and progress of the neural processes going on during the interval of training and test. In such an investigation he must ask himself such questions as: "Is the average net gain of my training group a measure of its progress?" "Does the absolute quantity of work done typify accurately the efficiency of my individual subjects in the mental ability studied?" The way in which the investigator answers these questions, the insight which he brings to the psychologic-statistical problem, the caution which he displays in adhering to a strict interpreta-

tion of his specific test-results—these will largely determine the validity of his investigation and its value for the advancement of practical school education.

B. *General Procedure.*

Our method may be roughly denoted as the classroom-experimental method in distinction to the laboratory method so often used in studies of transfer. The selected ability will first be trained (in as large a group of individuals as can be secured) by a semester's study in a regularly prescribed course of study in one of our higher curriculums. (This will be known hereafter as "the training course.") To determine efficiency in the ability before the training commences, a series of mental tests (known as "the test series") will be taken by the subjects—tests so designed as to offer a measure of efficiency in the ability under training. The subjects may be broadly classified as (1) "the training group" (those taking the training course of study), (2) "the control group" (those not taking the training course). Both groups will take the test series immediately before the commencement of the training series and again at its conclusion. The tests will be identical for both groups, and in both series will be taken under as nearly identical classroom conditions as it is possible to secure.

Granted that the tests do adequately measure efficiency in the ability studied, that the subjects of both the training and control groups satisfy the statistical criterion of a random selection (*e. g.*, in this case that they have been chosen at random by entire instructor-sections and not consciously selected for the presence or lack of any certain ability), that the mechanics of test-giving has been impeccable and that the classroom conditions have been maintained identical in all tests; that the attitude of

each of the subjects has been such that his best effort would be put forth on each of the tests—we may then feel confident that a comparison of the statistical results obtained from the training group with those obtained from the control group will offer evidence of a character available for the discussion of transfer of training or spread of improvement from one mental ability to other mental abilities.

C. *The Abilities Investigated.*

"To determine the effect of a semester's training in descriptive geometry upon the abilities of college students in the mental manipulation of spatial elements." By abilities in mental manipulation we shall mean the quickness and accuracy with which the verbal description of different types of spatial elements (whether plane figures or three-dimensional objects) raises in consciousness and maintains and manipulates in consciousness a corresponding mental picture or spatial "visual image."

For purposes of our discussion, therefore, the generalized ability in mental manipulation (it may be specifically of any one of the three types under consideration) has a threefold connotation; first, the focalization in consciousness of the image or the calling-up of the mental picture; second, the maintaining of this image or the concentration on this image in consciousness to the exclusion of other distracting elements; third, such manipulation of the image and its constituent parts as is necessary to the solution of the problem in which it appears as the basic element. For example, in our Test 5 (a test for geometric manipulation), after being told "to form a picture of objects described on the test sheet and to count the number of continuous straight lines which it would take to construct the object in space (and always to count each two coinciding lines as one)" the subject reads the follow-

ing description of a solid (among others on the list): "A square box with lid attached; lid open." Immediately he mentally manipulates the spatial elements representing a square box with a lid attached and open, concentrates on it to the exclusion of other elements trying to crowd into consciousness, and mentally goes around the box, top, sides, base, and lid, counting the lines as he goes till he reaches the total (15). Or again, in Test 2 (a test for non-geometric manipulation) he is asked to "mentally divide sixteen twenty-five by seven," doing all the work by mental manipulation and writing only the answer on the paper.

It should be noted here that the attempt will be made to measure mental manipulation, both quantitatively and qualitatively. To satisfy the criterion of *quantitative* measurement, the quickness or speed with which the mental manipulation takes place will be determined. For the *qualitative* criterion there is desired a measure of the accuracy with which the manipulated elements are correlated with the thing described. Moreover, we shall distinguish between three types of mental manipulation: first, mental manipulation of *geometrical elements,* or the ability to raise in consciousness and maintain and manipulate there, the spatial elements common to geometry and especially to descriptive geometry (including straight lines, plane figures, *e. g.,* triangles and squares, and solids with plane faces, *e. g.,* the wedge, cube and pyramid); second, manipulation of *quasi-geometrical* elements, in which the staight line is used in building up images of the various letters in a common rectangular alphabet (see Test 3); third, the manipulation of *non-geometrical elements, e. g.,* mental arithmetic exercises (problems in short division), or the formation of mental constructions using the letters in the spoken word.

CHAPTER III.

The Subjects.

The data on transfer presented in the following pages were secured from 413 students in the University of Illinois during the interval from September, 1911, to June, 1914. These subjects may be first broadly classified as (1) The Training Group, 326 subjects, students in the College of Engineering, who took both the Training Course (descriptive geometry) and the two test-series; (2) The Control Group, 87 subjects, 15 in the College of Engineering and 72 in the School of Education, none of whom took the training course, but all of whom took the two test-series.

The training group may be further subdivided into (1) the 1913 training group, 217 subjects taking the training course and test series in the interval February to June, 1913, the second semester of the collegiate year, (2) the 1914 training group, 109 subjects who took both the training course and the two test-series in the interval February to June, 1914. It will be noted that the subjects took their training course and both test-series under identical groupings of individuals and in the same classroom.

The control group in turn is subdivided into three subgroups, No. 1 being composed of freshmen engineering students, and Nos. 2 and 3 of students in the senior and junior classes registered in the School of Education. The 15 members of control group No. 1 were the only group of freshmen engineers taking work in the department of General Engineering Drawing who were *not* taking the course in descriptive geometry.

Thus, the selection of subjects was thoroughly random; that is, entire instructor-sections were used. The only

basis of selection was "taking descriptive geometry" or "not taking descriptive geometry."

It should be stated that a total closely approximating 600 individuals has taken either the first or the second test-series. The available number, however, was reduced by nearly one-third, owing principally to the fact that many individuals were absent from their classes at the time either one or the other of the test-series was given. Of those who took both test-series, all test-sheets have been used which were written in accordance with the requirements for each specific test. Only a negligible fraction of the test-sheets was discarded, however, on this account.

In order to trace the effect of general and specific training due to study in courses other than descriptive geometry, a separate grouping was made of all individuals in the entire training group. This grouping, known hereafter as *the training sub-sections,* was made on a basis of similarity of courses pursued during the training interval —that is, all individuals pursuing the same courses were placed in the same section. Thus, the training group was divided into seven sub-sections, each made up of students taking descriptive geometry and a given combination of the prescribed and elective courses of the College of Engineering requirements. Ninety-five per cent of the students taking the tests (in the training group) were freshmen in the College of Engineering, and, as the various engineering curriculums are made up largely of prescribed courses, only seven different combinations were required to include practically all the subjects.

The subjects of study listed for each section are as follows:

Group 1 (55 *individuals*).
descriptive geometry.
shop practice.
mathematics.
English.

Group 2 (119 *individuals*).
descriptive geometry.
shop practice.
mathematics.
foreign language.

Group 3 (8 *individuals*).
descriptive geometry.
shop practice.
mathematics.
chemistry or physics.

Groups 4 *and* 5 (20 *individuals*).
descriptive geometry.
mathematics.
architecture.
language or English.

Group 6 (60 *individuals*).
descriptive geometry.
mathematics.
science, English or language (includes all not taking shop practice).

Group 7 (37 *individuals*).
descriptive geometry.
all upper-class courses (*e. g.*, advanced science).
those having 2 English courses or 2 language courses.
those having no mathematics.

It will be noted that the training course, *descriptive geometry, is the only course common to all of the training sub-sections.* Of the other courses, mathematics (algebra, trigonometry and analytic geometry have been grouped together for the preliminary analysis of the cause of possible gain on the part of the training group) occurs in all groups except No. 7. As will be brought out in detail later, this is justified by the fact that 85 per cent of the individuals pursuing studies in mathematics are found in the course in analytic geometry. Shop practice courses (primarily practical work in the forge and machine shop) occur in Sections 1, 2, and 3. Groups 4 and 5, which contain together but 20 individuals, have been combined to study the possible effect of training in architecture. Groups 1, 2, and 3 are alike in that they contain students of descriptive geometry, shop practice and mathematics. They differ only in the fourth course included, English in Group 1, modern languages in Group 2, and science in Group 3. Group 6 contains the students having no shop practice, and in Group 7 have been brought together all those having no mathematics, all upper classmen and all those having two or more courses of a kind. Further discussion of our theory of grouping these training sub-sections will be found in the chapter on statistical results.

CHAPTER IV.

The Training Course: Descriptive Geometry.

Descriptive geometry is a prescribed course of study of the freshman or sophomore year in the engineering curriculums of one hundred and thirty-one higher educational institutions of the United States, offering a four-year degree-course in engineering. The writer's study of "The Historical Development and Present Status of Technological Education in the United States" (1914) shows that descriptive geometry has become a standard prerequisite for advanced courses in engineering drafting and design. Typical practice now offers it as a second semester course of the freshman year, and it is found in all engineering curriculums, civil, mechanical, electrical, etc. (the University of Illinois follows this practice), generally accompanying or immediately succeeding a semester's course in the elements of mechanical drafting.

Its fundamental purpose is to build up for the student a method of picturing solid objects (*e. g.*, machines, like engines, dynamos, their constituent parts, and the diverse materials used in engineering construction of all sorts) on one plane, called the plane of projection, and coincident with the drawing paper. Just as the component elements of solids may be obtained by successive differentiation of solids themselves, so we may regard any solid as analysable into its component planes, straight lines, and points. Thus, we find that, theoretically, we may regard a straight line as a series of points, a plane as a series of straight lines and a solid as a series of planes. Since the goal is to be able to picture solid objects on one plane, the intermediate steps must obviously build up a method of pic-

turing the spatial position of points, lines and planes, first separately and then in combination with each other. That is exactly what the course does. It is fundamentally a developmental course of problems, in which each problem provides a method of representing the spatial position of some desired combination of points, lines, planes or solids on the plane of the drawing paper. Thus, it should be emphasized that descriptive geometry is dealing throughout with the spatial representation of things, and that the correct understanding of the solution of each problem in it necessitates a mental construction of the things in question.

Since its introduction into this country (by Col. Claude Crozet as a part of the engineering curriculum of the United States Military Academy in 1807), descriptive geometry has been regarded as having considerable applied value for engineers and draftsmen. During the nineteenth century development of engineering curriculums in our higher technical schools this point of view became so strongly intrenched that nearly all teachers are teaching it for its value as an applied science or as a branch of applied mathematics. However, the Department of General Engineering Drawing in the University of Illinois regards the course as *a branch of pure mathematics*. This fact is particularly pertinent to our present investigation. Since the course is regarded, primarily, as a "disciplinary" course, each successive problem in the course is designed (among other things) to function as a definite bit of discipline in spatial picturing. The majority of the instructors in the department regard a stage of "visualization" as fundamental to the effective presentation of each problem. "See the problem in space;" "Picture the lines and planes in your problem;" "Image the problem"—these are exhortations commonly heard in our

classrooms. This constant "drill in visualization," coupled with the use of mechanical devices for furthering it, has led to the belief that four months' study of descriptive geometry must fulfill a function in "training the imagination."

In the training course, the subject-matter of the course has been made absolutely uniform for the entire department. Week by week throughout the four months, the 300-odd students, grouped in 12 teaching sections under 6 different instructors, are pursuing precisely the same exercises in the classroom (the solution of blackboard problems) and the same practice in the drawing room (solution of the same type of problems on drawing paper). The only possible variation in the work is in the method of presentation, and even here at least four of the six instructors use very similar methods.

CHAPTER V.

The Tests.

1. *Preliminary Tests.*

To secure tests which would adequately measure efficiently in the mental manipulation of spatial elements, a preliminary investigation was conducted during the summer of 1911 and the first semester of the following college year. Tests were designed, discussed with colleagues and advisors, and tried out with some 40 students, principally students in the writer's classes in descriptive geometry and mechanical drawing. This preliminary experiment was of the laboratory type. The tests were orally presented to individual students in the writer's office, time was taken by a stop watch, and the answers to problems were given orally by the subjects and recorded by an assistant. From their original form these preliminary tests went through a process of change to adapt them to the needs of the problem. In the design of these tests two considerations were kept in mind: (1) the tests must not be so long as to permit within themselves large effects of practice, (2) the solution of none of the problems must depend on previous training of a specific type, *i. e.,* they must be of so "general" a nature that all college freshmen will be able to solve them. Owing to the constant changing of the tests as given to the different subjects, the results of this preliminary investigation are not comparable, and are of value to the final investigation only as aids to the perfecting of the tests given below. For that reason, neither the preliminary tests nor their results will be presented here.

2. *The Final Tests.*

The tests as taken by the subjects may be grouped as: (1) tests for the manipulation of non-geometrical elements (Tests 1, 2, and 6), (2) tests for the manipulation of quasi-geometrical elements (Test 3), (3) tests for the manipulation of strictly geometrical elements (Tests 4 and 5). These tests are reproduced below in the exact form, but not in the order in which they were taken by the subjects. The order of taking the tests will be explained in the chapter on "Securing the Data." The answers regarded as correct by the judges who scored the tests are given in parenthesis after the problems. In addition to the tests themselves, we give below a statement of the purpose of each. This statement does not include the mental functions believed to be common to all the tests. These are discussed in detail after the exposition of the tests themselves.

(*a*) *Tests with Non-Geometrical Elements.*

Test 1. Time given: 60 seconds.

1	Divide eighty-one by seven..................	($11^4/_7$)
2	Divide seventy-eight by four..............	($19^1/_2$)
3	Divide ninety-five by seven................	($13^4/_7$)
4	Divide fifty-nine by four....................	($14^3/_4$)
5	Divide seventy-one by three..............	($23^2/_3$)
6	Divide eighty-two by three................	($27^1/_3$)
7	Divide seventy-nine by six................	($13^1/_6$)
8	Divide eighty-nine by six..................	($14^5/_6$)
9	Divide ninety-six by seven................	($13^5/_7$)
10	Divide ninety-two by six..................	($15^1/_3$)

Test 2. Time given: 90 seconds.

1	Divide eight sixty-two by three............	($287^1/_3$)
2	Divide seven ninety-five by four...........	($198^3/_4$)
3	Divide four seventy-eight by three.........	($159^1/_3$)
4	Divide seven fifty-one by two..............	($375^1/_2$)
5	Divide sixteen twenty-five by seven........	($232^1/_7$)
6	Divide thirty-four fifty-two by nine........	($383^5/_9$)
7	Divide ninety seventy-one by four..........	($242^3/_4$)
8	Divide twenty-seven ninety-one by eight...	($348^7/_8$)
9	Divide fifty-seven forty-three by six........	($957^1/_6$)
10	Divide eight twenty-seven by six...........	($137^5/_6$)

Tests 1 and 2 are regarded as giving measures of the efficiency of the manipulation of spatial elements of a purely non-geometrical type. The solution of the problems contained in the tests also involves skill in arithmetical fundamentals. There are several reasons that justify us in believing that any training in arithmetical fundamentals received during the training interval need not be considered as a source of error: (1) the skill is involved in both the February and June test series and on the part of both the training and control groups; (2) the subjects are all adults, having completed at least one semester of college work and, being registered in the same department of study, have in the long run undergone approximately the same previous training in mental arithmetic; (3) the effect of such training, in creating inequalities in skill and fundamentals, coming, as the training does, in the elementary grades, would have long since been smoothed out through the neglect of the function; (4) according to the statement of the students, those individuals who through practical office work have had special training in mental arithmetic are so few as to have no

material effect on the results. The memory factor was practically eliminated by requesting the students to write the digits of the answer as rapidly as they pictured them.

Test 6. The word "Material" test.

Time given: 300 seconds.

The subjects were required to picture the letters of the word "material" to form from them as many new words as possible and to write these in order on a blank slip of paper. To maintain the efficiency of the test as a test for the mental manipulation of spatial elements, each word was hidden from view as soon as written. To indicate approximately the interval between the obtaining of the words, the subjects drew lines under the last word written as the successive minutes of elapsed time were announced by the investigator.

In the solution of Test 6 it is possible to bring to bear—either separately or in combination—at least five mental functions, viz., visual imagery, auditory imagery, systematic method (or "system"), reorganization of the letters previously used and ideational or perceptual devices. It would naturally be questioned: "How can the mental functions active in producing the resultant efficiency as shown by this test be credited properly for their respective participation in the completed product?" The validity of the test will be discussed in detail in Chapter VIII, "Scoring the Tests."

(b) *Test with Quasi-Geometrical Elements.*

Test 3. Straight-line alphabet test.

Time given: 75 seconds.

cat | pen | jury | does | rank | this | why | girl | about | lapse | him | stead |

The problem: "picture each word on this sheet in the straight-line alphabet shown on the blackboard [reproduced below], and write below each word the total number of continuous straight lines (strokes) which would be required to print the word using this style of letter."

Test 3 is believed to be a measure of ability in the mental manipulation of quasi-geometrical elements (plane figures—no three dimensional objects). Memory plays very little part (it is believed to be negligible) in the solution of the test problems, as they are made up of very short words.

(c) *Tests with Strictly Geometrical Elements.*

Test 4. The Painted Cube Test.

Time given: 80 seconds.

A three-inch cube, painted on all sides, is cut into one-inch cubes.

1. How many one-inch cubes have paint on three sides?
   Answer........(8).
2. How many one-inch cubes have paint on two sides?
   Answer........(12).
3. How many one-inch cubes have paint on one side?
   Answer........(6).
4. How many one-inch cubes have paint on no side?
   Answer........(1).

Test 5. Geometrical Objects Test.

Time given: 90 seconds.

The problem: "form a mental picture of each object and count the number of straight lines which it would take to construct each one in space."

1. A wedge......Answer........(8 or 9).
2. Four triangles attached to a square, bases coinciding with the sides of the square. Answer........(12).
3. A square box with lid attached. Assume lid as open and having no thickness. Answer........(15).
4. Three squares attached to an equilateral triangle, one side of each square coinciding with a side of the triangle. Answer........(12).
5. A triangular pyramid resting on a triangular prism, bases coinciding. Answer........(12).
6. State the number of *bounding lines only* in a window with a triangular top and a square bottom. Answer........(5).
7. A wedge resting on a cube, edges coinciding. Answer........(17).
8. State the least number of lines necessary to draw a square window divided into four panes. Answer........(6).

Tests 4 and 5 are regarded as measures of ability in mental manipulation of strictly geometrical elements. A three-dimensional object was used in Test 4, and both two and three-dimensional figures in Test 5. The diversity of the tests and the short time given for the solution of each one are believed to prevent effectually the building up of a definite method of solution through reasoning processes (organization, etc.). In other words, it is believed that a solution of each of these tests is confined to (1) a mental picturing of the object, (2) the counting of the various lines and surfaces, (3) the writing of the various answers.

There are certain functions which are common to the solution of most, if not all, of the tests. These functions are present, of course, in the solution of both the first and second test-series, and are all functions to which elementary and secondary school education has contributed training to such an extent that our training course is regarded as having no further effect. In other words, the efficiency of these functions is considered as equivalent in both the February and June test series.

Those functions active in all tests are (1) *accuracy* in recording the solutions to problems (*e. g.,* writing the answers to short division problems, writing the number of straight lines in certain straight-line letters or the number of lines in certain geometrical shapes), in brief, the ability to set down accuractly the result which has been derived; (2) *quickness* in recording results to problems. Both accuracy and quickness are present in the two series, and the work performed is so slight in extent, and the interval between the taking of the two test-series is so great, that the functions are regarded as negligible in discussing the validity of our tests as measures of improvement in mental manipulation.

Mental functions involved specifically in the solutions of Tests 3, 4, and 5 include accuracy in counting lines or plane surfaces and quickness in counting lines and plane surfaces.

In addition to the above functions, careful attention was given to the activity of a common factor affecting both test series—the attitude of the subjects. The investigator and his assistants used great care in determining the status of this factor in each section in which tests were given. The attitude of the 413 subjects whose re-

sults have been used in this investigation may be denoted as one of serious-minded interest in solving each test to the best of their ability. The results of three subjects whose attitude seemed careless or antagonistic were discarded.

CHAPTER VI.

SECURING THE DATA.

The data of this investigation were secured at the beginning and close of the second semester of the two college years, 1912-13 and 1913-14. The 1913 training group took its first test-series between February 2 and 6 and its second between May 25 and 27. The 1914 training group took its first series February 12 to 14 and its second between May 27 and 30. The control group No. 1 took its first series February 13 and its second June 2; control groups 2 and 3 took the first series February 17 and its second May 28. Thus, the training was for all practical purposes identical for all groups, averaging about 15 weeks. In the case of each training group the first test-series was taken during the first week of study in descriptive geometry and before any of the training in mental picturing was begun. The amount of training given each section of the two training groups was therefore identical.

The tests were taken by the sections in their customary classroom and at their regular weekly and daily time of meeting. In each case the instructor in charge of the section prepared the way for the taking of the tests by brief introductory remarks. Following these the writer conducted the taking of the tests by the section. All tests, both February and June, 1913 and 1914, were thus conducted by the one investigator, so as to secure as nearly perfect uniformity as possible. The steps involved in the conduct of the tests included certain general and certain specific instructions.

(a) *General Instructions.*

There was given the following brief explanation of the nature and purpose of the tests. Special care was taken to secure the undivided attention and interest of every individual member of the class:

"You men may know that in many of our larger universisities throughout the country, like Chicago, Wisconsin and Columbia, there are being carried on investigations of an educational nature. By performing experiments in their classes, teachers are trying to find out what they can about the abilities of students, not only in the colleges, but in the high schools and elementary schools as well. To do so they are asking pupils in their classes to solve simple test-problems which have been devised for that purpose.

"Now, we are carrying on such an investigation in this department of the University of Illinois, and would like to ask you to help us in it, as Mr. ——— (instructor of the section) has kindly permitted us to use about half an hour of your time this morning, instead of spending it on descriptive geometry. You will understand, of course, that this help will be entirely voluntary on your part, and that your interest in, and careful attention to, the tests will be a very great favor to us. Of course, these tests have nothing to do with your standing in descriptive geometry, and I may say that nobody in the department except myself will see your results. So please give your very best effort to the solving of the problems in each test. Remember we are trying to determine two things in each of the tests—your accuracy and your quickness. As you take up each test, work as many problems as you can and work each as accurately as you can. Each of the tests is a time test. Before we take up each test I will make a

careful explanation of its nature and its purpose, telling you exactly what to do, also telling you how much time you are to be given. Then we will hand you the test paper and ask you to keep it printed side down and to write your name on the back at once At the signal *"go,"* please turn the paper, read the directions and the statement of the problems and work as many of them as you can and each as accurately as you can. At the signal *"stop,"* please stop instantly and do not write anything on the paper thereafter. If you complete the entire test before hearing the signal "stop," please hold up your hand instantly, and I will name the time (in seconds) which you have taken. Please write this time on your paper. As soon as time is up, please pass the papers down to the right, and we will proceed to the next test. We will now take up the first test."

This general explanation was given to each section that took the tests in the two years, with a very slight change in the references to descriptive geometry in the case of the control groups. Care was taken with all sections to give the same emphasis to the main points of the explanation. It is believed, from consultation with the instructors in charge of the sections, that this was done satisfactorily. Following these general directions the tests were at once taken up in order, and certain specific directions given for them in all the sections. These directions for the various tests are reproduced below in the order in which the tests were taken by the students. For the tests themselves consult Chapter V.

(b) *Specific Instructions for the Several Tests.*

First test (straight-line alphabet test, No. 3). While the test papers were being handed out the straight-line alphabet (reproduced in Chapter V) was put on the black-

board, with this explanation: "Please note carefully the formation of the letters in this alphabet. Notice that they are all capitals, that there are no curved lines, and that each letter has been made with as few straight lines as possible. Are they understood?" The alphabet was then erased. "In this test you will find on the test sheet a list of short words. We want you to form in your mind a picture of each word, beginning at the left of the sheet. Count the number of lines which it would take to print each of these words if you used the alphabet which you have just seen on the board. Be sure to use no other letters than those used in the style just shown you. Write the total number of lines which you obtain for each word below the word. Do not write or print the words themselves. If you finish before the signal "stop," be sure to hold up your hand instantly and write the time that I give you on the back of your paper. Do not write anything after the signal "stop." Is your name on the back of your paper? Ready? Go!"

At the completion of the test each section was asked to print on the back of the sheet the style of alphabet that had been used in the problem. Thus, the accuracy of each subject was checked in the scoring of the tests.

Second test (short-division test, three operations, No. 1). "In this test you will find on the paper a list of ten examples in short division. At the signal we wish you to solve these in your head, but write nothing on the paper but the answer. You may write the answer as a whole number and a fraction, thus [illustration on the blackboard]. Instead of retaining the different digits in the answer until you have them all, please write them on the paper as fast as you get them, thus [illustration with problem on the board]. Work as many problems as you can, and work each as accurately as you can. Do not

write anything on the paper after you hear the signal "stop." Is your name on the paper? You will be given 60 seconds in this test. Ready? Go!"

In Tests 1 and 2 the 1913 training group was required to turn the paper after reading each problem and to write the answer on the back. The 1914 training group and the control group were both allowed to write the answer on the front of the paper.

Third test (geometrical objects test, No. 5). "On the other side of this paper you will find a list of names of objects, some of which are plane figures and other three-dimensional objects. In this test we want you to form in your mind (beginning at the top of the paper with the first object) a picture of each object, and to count the number of straight lines which would be necessary to construct that object if it were standing alone in space. Remember to count all the lines in the object, not simply those which you can see from one or two sides as in a perspective drawing. Count the base-lines, the vertical lines, the inclined lines, if there are any, and the top lines. If, in any object, two lines coincide, please count those two lines as one. For instance, suppose I had a large rectangular block of wood on this table, so that the edges of the block exactly coincided with those of the table top. Then I would count these two edges as one, these two as one, etc., all around the table. Is that point understood? Write the answer in the place provided after each problem. Do not write any part of an answer after the signal "stop." Your time in this test will be 90 seconds. Is your name on the back of the paper? Ready? Go!"

At the conclusion of the test the class was requested to turn the papers over and to make a small sketch on the back to show the number of lines counted for each object worked. This served as an admirable check on the accuracy of the scoring of the papers.

Fourth test (second short-division test, four operations, No. 2). "This test is another exercise in short division, similar to the one you just had. The directions are precisely the same as in that test: begin at the top of the paper, work as many problems as you can, as accurately as you can, and write only the answer on the paper. Write the answer as a whole number and a fraction, as before, and write the digits in the answer as rapidly as you get them. Since these problems are a little longer than the others, we will give you 90 seconds on this test. Do not write a digit after you hear the signal "stop." Is your name on the paper? Ready? Go!"

Fifth test (painted cube test, No. 4). "There is no explanation in this test other than you will find on the paper. Read very carefully the directions given there. If you finish before time is up, hold up your hand instantly and write on your paper the time that I give you. You will be given 80 seconds on this test. Is your name on the back of the paper? Ready? Go!"

Sixth test (the word "Material" test, No. 6). Blank sheets of paper were handed to the class, followed by this direction: "In this test I am going to pronounce and spell carefully a word. I want you to notice very carefully exactly what letters are used in this word, and the way in which they may be repeated. As soon as you get the word, please form as many new words as you can from the letters of the given one and write the words as rapidly as you form them on the sheet. In any one of your words use only the letters of the given word, and repeat in any one word only those letters which may be repeated in the given word. As rapidly as you write the words, please hide them from view, thus [illustrate]: do not under any circumstances refer back to your list. If you have forgotten whether you have written a certain word, write it again

rather than look back. Hide words by drawing the paper over the arm of your chair. You will be given five minutes on this test, and I will tell you the time by minutes. As I name the successive minutes, please draw a line across the paper under the last word which you have written, and go on to the next word. I shall say at the end of each minute, "Draw a line, first minute, second minute, etc. Is your name on the paper? Ready for the word? This word is M-A-T-E-R-I-A-L, Material."

At frequent intervals the investigator mentioned to the class that they must hide the words from view. On the whole, this direction was followed.

(c) *Miscellaneous Points.*

From the preliminary investigation the tests had been timed so that the average student would finish about half of the work given on each test sheet at the time of taking the first test-series. The time given proved to be adjusted in this proportion for practically all sections, and few subjects finished before the signal "stop" was given. It was arranged that those who did finish should write the elapsed time on their papers, and thus there was given a further check on their speed. In the case of the fourth and sixth tests, those individuals who were familiar with the problems given stated this on their papers. Their scores were not included in the tabulation of results.

In order to keep any knowledge of the investigation from the students in other sections of the department, the members of each section were asked not to discuss the tests in any way. Those in succeeding sections were asked if they heard anything of the tests. The replies indicated that our request had been complied with by the subjects who had already taken the tests, and that each section came to the tests with the same attitude toward them.

At the end of the June test series introspections were obtained from practically all sections on the working of each of the tests. Since these subjects were entirely untrained in introspective analysis, these were regarded as rough aids only in the interpretation of our tabulated scores. The method of securing valid introspections on the mental processes involved in the solution of the various test problems is explained at length in Chapter X, "The Theory of Transfer."

CHAPTER VII.

Scoring the Tests.

All the papers were scored by the writer. Tests 1 to 5, inclusive, of the 1913 training group were scored in October, 1913, the remaining tests in the summer of 1914. All who took both test-series were first numbered according to the system used in the Department of General Engineering Drawing, University of Illinois; the 1913 training group formed Nos. 1 to 316, the 1914 training group Nos. 400 to 499 and 600 to 609, the control group Nos. 501 to 588. These numbers were used on all papers, and the subjects were known thereafter by number.

The method of scoring each test is given in detail below:

Tests 1 and 2 (short-division tests). Speed was scored as the total number of digits recorded in the answers to all problems worked in the test; accuracy as the total number of these digits found to be correct. Thus, a score of 27/19 given in a paper was later tabulated as 27 (mental operations) attempted and 19 worked correctly. Efficiency is expressed as the per cent of total operations correctly worked. These problems were orginally scored by merely stating the number of problems worked and the number right. It was perceived that this method was subject to large inaccuracies and the method was further refined, so that the score of the test should indicate as closely as possible the true status of the accompanying mental process.

Test 3 (straight-line alphabet). Speed was scored as the number of words attempted, accuracy as the number of words for which the correct number of lines was stated. A positive check was secured by having the student print on the back of the test sheet the exact letters that he used.

Test 4 (painted-cube test). Speed was scored as the number of problems attempted, accuracy as the number of problems for which the correct number of painted faces

was stated. It is recognized that the measure of efficiency of correlated mental processes is here very much less refined than in the other tests. There was no way found, however, by which greater refinement could be obtained.

Test 5 (geometrical-objects test). Speed was scored as the number of problems attempted, accuracy as the number of problems for which the correct number of lines was stated. As in Test 3, a positive check was secured by having each subject make a drawing of the objects worked.

Test 6 (the word *material* test). The number of words obtained in each of the successive minutes of the test, and the total number of words obtained were recorded on each paper. All repetitions of words in a given paper and all words using letters not in the given word were disregarded.

The method of scoring to determine the specific psychological processes, or functions, involved in the solution of Test 6 was first standardized by consultation of four adult judges, all of whom had had psychological training. Sample papers were given to these judges, with the request that they work through the list of words and analyze them carefully to determine the psychological functions which might have contributed to the obtaining of the lists. Following this, joint consultation of the judges was held to discuss the results and criticize each other's methods. Five possible mental functions were agreed upon which may have contributed to the obtaining of the words on the list. These were visual imagery, auditory imagery (or kinesthetic imagery), reorganization of letters previously used, systematic method (*e. g.,* that of consciously searching for words beginning with the same letter), and ideational or perceptual processes. A composite method of scoring the mental process behind the formation of each two consecutive words was then made up arbitrarily by the writer and applied to the papers of all subjects. To

illustrate the working of this method a sample list of words and its scoring is given below. In this list "x" indicates the specific mental function or functions believed to be active in the formation of successive words.

Key: A = auditory imagery; O = reorganization of letters; S = system or systematic method; V = visual imagery; I = ideational processes.

		A	O	S	V	I
	mat........................					
	at........................	x	..	..	..	..
	tale........................	..	x	..	x	..
	ale........................	x	..	..	x	..
	male........................	x	..	..	..	..
	rail........................	x	..	..	..	..
1′	tail........................	x	..	..	..	..
	ream........................	..	..	..	x	..
	realm........................	..	x	..	x	..
	tear........................	..	..	..	x	..
	tire........................	..	..	..	x	..
2′	ire........................	x	x	..	..	..
	it........................	..	..	..	x	..
3′	rite........................	..	..	..	x	..
	liar........................	..	..	..	x	..
	lie........................	..	..	..	x	..
4′	rile........................	..	..	..	x	..
	ear........................	..	x	..	x	..
	are........................	..	x	..	..	..
	air........................	..	..	..	x	..
5′	aerial........................	..	..	..	2 x	x

Naturally this method gives no absolutely precise measure of the extent to which the various mental functions have been operative, but judging from the outcome of its application to more than 16,000 words, a close approximation to the participation of the various functions was secured. It seems assured, at least, that Test 6 is largely a test for visual manipulation, as the scoring indicates that over 80 per cent of the words were probably contributed by this function. Again, that systematic method and ideational processes do not play a large rôle is clear from our later tabulations (see Tables D and E).

CHAPTER VIII.

TABULATION OF THE DATA.

The procedure followed in tabulating our data may be outlined as follows:

1. *The Original Record Sheets.*

The transferring of the results of individual papers to original record sheets was the first step in the work of tabulation. For Tests 1 to 5, inclusive, these sheets were organized in the manner illustrated herewith. One record sheet was used for each section. The sample given shows a sample entry for one subject in one test.

Test No........... ....Group
Section No......... ....Year

No of Subj.	Sub. Trg. Sec.	Grd. D. G.	Score in No. problems attempted. Feb.	June.	G.	E.	L.	Per cent. gain. G.	T.
94	6	78	24	30	6	..	..	25	..

Score in No. of problems right. Feb.	June.	G.	E.	L.	Per cent. gain.	Efficiency in per cent. Feb.	June.
24	30	6	..	..	25	80	100

(Sub. Trg. Sec. means sub-training section of the training group; Grd. = grade in descriptive geometry; G. = gain; E. = even record (constant); L. = loss; T. means time.)

For Test 6 there was recorded for both the February and the June test series (1) the number of words formed in each minute, (2) the total number of words, (3) total gain, (4) the per cent gain, (5) the number of words formed by each of the five mental processes, A, O, S, V, and I.

These original record sheets thus give for each subject in the investigation (1) the absolute quantity attempted and the amount performed correctly in both test-series, (2) the gain of each subject, both in absolute numbers and in per cent, (3) the efficiency attained in both test-series. Since the data of these original records are all embodied in summary form in Tables A-1, A-2, and A-3, we have not reproduced here the 110 original tables for Tests 1 to 5, inclusive. No data of any importance to a thorough understanding of the study have been omitted.

2. *Tables for the Comparison of Absolute Attainment of Individuals and Groups.*

Tables A-1, A-2, and A-3 give the efficiency of each subject in Tests 1 to 6, inclusive.

Tables B-1 to B-20, contingency tables for Tests 1 to 5, give the number of subjects making certain number of attempts and getting a certain number of these attempts correct. These give also (for training and control groups separately) the median number of problems attempted and the median number right. These tables offer a second method of comparing the work done by the two groups and thus of determining possible resultant progress in imagery due to the training course.

Table C gives for Tests 1 to 5, inclusive, a summary of the number of problems (or operations) worked and the median gains made by training and control groups.

Table D gives for Test 6 the aggregate and average number of words obtained in both test-series and for both groups.

Table E gives the aggregate and average number of words obtained by each of the five mental functions involved and the corresponding per cent gain in various groups.

3. *Tables for the Determination of Relative Gains Made by Training and Control Groups.*

Tables F-1, F-2, and F-3 give for Tests 1 to 6, inclusive, the per cent gain made by each subject.

Table G gives for Tests 1 to 6, inclusive, the number and per cent of subjects who gained in both "Attempts" and "Rights."

Table H gives a summary of the aggregate and average gains in Tests 1 to 5, inclusive.

Table I gives the number of subjects who gained in various propositions of all tests taken in mental manipulation.

4. *Tables to Aid in Tracing the Cause of Greater Gain on the Part of the Training Group.*

Table J gives for Tests 1 to 5, inclusive, a summary of the efficiencies and per cent gains of the subjects as grouped in the seven training sub-sections.

CHAPTER IX.

STATISTICAL RESULTS OF THE TABULATION.

From the foregoing, it will be seen, there are five stages in our statistical attack upon the problem before us, viz., (1) the transference of the test-sheet scores to our original record sheets, (2) the determination of the absolute quantities of work done by our training and control groups, *i. e.*, of the absolute efficiencies in the mental functions under observation, (3) the determination of the relative gains made by the training group as compared with that of the control group, (4) the isolation of the cause of a residual gain in favor of either group, (5) the correlation of efficiency in the mental functions involved in these tests.

A study of the quantitative aspects of these problems should lead to the answer to such questions as: What is the initial and final attainment of various individuals? What is the relative efficiency of the various groups? What is the relation of speed to accuracy in resultant efficiency of the various tests? What is the typical gain of the individuals? of the groups? Which group gains the more? In what proportion of all the tests taken do the various subjects gain? If there is a gain in certain tests, is that of the training group greater or smaller than that of the control group? Can our statistics aid in isolating the cause of the residual gain?

1. The Efficiency of Individuals and Groups.

Those interested in the study of the detailed records of this investigation will find in Tables A-1, A-2, A-3 the per cent efficiency attained by each of the subjects in both

February and June test-series in Tests 1 to 6, inclusive. The figures given there represent the per cent of the total oprations or problems solved correctly by each subject. Thus, in Test 1, of a total of 30 operations, an efficiency of 90 per cent corresponds to 27 digits right. In Test 6, however, the figures given represent the number of words formed.

It must be recognized that the evidence of efficiency is twofold: speed and accuracy must both be considered. In other words, how many problems did each subject attempt, and how many did he solve correctly? The latter aspect corresponds to "efficiency" as expressed in our tables.

More accurately to picture the status of efficiency among the subjects, both individually and by groups, 20 "contingency" tables are presented herewith, Tables B-1 to B-20. (The writer is adapting Professor Karl Pearson's term "contingency" to his use in this immediate problem.) The design of a contingency table of this type has revealed a method of mathematically and pictorially representing absolute attainment in both "Attempts" and "Rights" for each of the tests. Analytic and comparative study of these tables will give a definite idea of the efficiency of the training and control groups.

To complete the "type picture" of the progress of the two groups we provide summary-table C, which gives the median number of problems (or parts of problems) attempted and right, with the accompanying gains. We are now in a position to discuss the amount of work done by both training and control groups in Tests 1 to 5, inclusive, recalling here the grouping of these tests: 1 and 2 non-geometrical, 3 quasi-geometrical, 4 and 5 strictly geometrical. A glance over the February efficiencies reveals the fact that the training group maintains a slightly higher

initial efficiency than the control group in the non-geometrical tests and in the quasi-geometrical test, and that the initial efficiency of the training group in the strictly geometrical tests considerably exceeds that of the control group. That is, we find that the initial efficiency of the training group is markedly larger in all of the tests. (The training group is made up of engineering freshmen, whereas the members of the control group are principally juniors and seniors in general arts courses.)

But what proves to be the status of efficiency in the two groups at the second test taking? Examination of the June efficiencies shows that the efficiency of the training group exceeds that of the control group in the non-geometrical tests, the quasi-geometrical test and in the strictly geometrical tests. The gain in "Rights" is regarded, throughout this investigation, as the most valid measure of the relative progress in these tests. It is found that in "Rights" the gain for the training group (taken as a whole) is larger than the gain of the control group in all tests. The residual gain in favor of the training group is for each of the five tests, respectively,—0.9 per cent, 15.78 per cent, 20.4 per cent, 14.0 per cent, 48.5 per cent. Again, if we consider the number of problems attempted as a partial check on the relative efficiencies of the two groups, the training group proves to be superior in all tests but Test 4. Here the residual gain is for the five tests, 1.4, 5.5, 6.4,—8.2 and 15 per cent, respectively. In the case of Test 4 (the painted-cube test) a large number of subjects finished the test before time was called. Their elapsed time was recorded on the original record sheets. No provision has been made on the contingency tables for time factor; otherwise, Test 4 would reveal a residual gain for the training group as in the other four tests.

For the purpose of roughly indicating the *tendencies* in the three types of mental manipulation, the efficiency obtained by each group in Tests 1 and 2, and in Tests 4 and 5 will be averaged. An interesting progression results. The residual gain for the training group over that of the control group is now, for the non-geometrical tests, 7.44 per cent; for the quasi-geometrical test, 20.4 per cent, and for the strictly geometrical tests, 31.25 per cent. The same progressively increasing gain in favor of the training group exists in the results for "Attempts;" 3.47 per cent for the non-geometrical tests, 6.4 per cent for the quasi-geometrical test, and 11.6 per cent for the strictly geometrical tests.

The results for Test 6, another test with non-geometrical elements, give additional data on this question. Turn to Table D. In the February test the training group obtained an average of 19.1 words, the control group 21.3 words; in the June tests the figures are 23.90 and 23.80, respectively. Thus, the relative gain of the training group is 24.87 per cent, and of the control group 11.5 per cent. This is of interest, as it leaves a residual gain for the training group of 13.37 per cent. The solution of Test 6, however, involves five possible functions, and the general results are complicated by the possible contributions of reasoning functions, like organization, systematic method or ideational or perceptual processes. Table E gives the specific data that are needed at this point; that is, the number of words formed by each of the five psychological methods. Using now the data for words formed by visual manipulation alone, we find that the training group gains 22.8 per cent of its initial efficiency, and the control group 15.4 per cent—a residual gain for the training group of 7.4 per cent. Test 6, then, at least to a small extent, reinforces the results obtained from Tests 1 and 2, namely,

that the training group shows a progressively greater gain over the control group as one proceeds from the non-geometrical field of mental manipulation to the strictly geometrical field.

2. *Determination of Relative Gains Made by Training and Control Groups.*

Thus far there has been considered in the tabulations the entire number of subjects in the two groups. It has been found that there is a difference in the amount of work done by the two groups in their initial performances; that this difference is constantly in favor of the training group, (most pertinent to the problems of this investigation) that, regardless of the greater initial efficiency of the training group, the gain of the training group is greater in every test than is that of the control group. So much have the tables of initial and final efficiency contributed.

We now take up another phase of the problem: *the status of the gainers.* New questions have now to be answered: (*a*) How many individuals gain in each group in each test? (*b*) How large is their gain in each test and in each of the types of mental manipulation? What is the average gain of each group in each test? (*c*) In how many tests do the gainers gain? Are they consistent gainers, gaining in all types of tests? What is the typical progress of the training group compared with that of the control group?

(*a*) *How Many Individuals Gain in Each Test?*

Since the two groups do not contain the same number of individuals, the absolute number of gainers is not of interest. In Table G, however, comparable data are found

for the two groups, viz., the per cent of each group gaining in each test.

"Attempts." From the figures of the status in "Attempts" of all training subjects there is found a remarkably constant percentage of gain; this is, for each of the five tests in order, 69.2, 64.2, 64.6, 63.7, and 66.0 per cent. Here the extreme variation is only 5.5 per cent. The "Attempts" recorded of the control group reveals the same small variation, but a constantly smaller percentage of gainers than in the training group. The figures for the five tests, respectively, are: 47.6, 45.3, 63.6, 43.7, and 45.8 per cent.* On the whole, then, we have two consistent groups: approximately 45 per cent of the control group gain in Tests 1 to 5, inclusive, and approximately 65 per cent of the training group gain in the same tests. Turning our figures of superiority of the training group into per cent of the number of gainers in the control group, we find that our training group had 44 per cent more gainers, or nearly half again as large a proportion of gainers as did the control group.

"Rights." As a more adequate test of efficiency, the evidence is here still more conclusively on the side of the training group. The per cent of gainers in this group for each of Tests 1 to 5, inclusive, is 70.0, 67.8, 57.6, 55.3, and 62.5 per cent; for the control group, 47.6, 39.5, 44.2, 37.8, and 35.3 per cent, respectively. Thus, in the case of "Rights," there is a distinct difference in ability to gain in the various types of mental manipulation. A wider variation is found in both groups, 18.7 per cent in the training group and 12.3 per cent in the control group. But pertinent to this inquiry and reinforcing the evidence

*In spite of detailed study of the larger gain in "Attempts" in Test 3, the author can offer no complete explanation for it. It seems to be due to a larger gain throughout the group and not to abnormal gains of any one of the control sub-groups.

as noted above, we have more than half again as many gainers in the training group as in the control group; the average of the transposed per cents is 56.2 per cent. Especially significant to the solution of the problem is the fact that both in non-geometrical tests (1 and 2) and in the strictly geometrical tests (4 and 5) there are nearly two-thirds again as large a proportion of gainers in the training group as in the control group.

(b) *How Large is the Gain of Individuals and Groups?*

If familiarity with the tests and practice-effect increase attainment in this investigation, it might well result in a large proportion of the control group showing a gain in the June test series. It actually did result in from 35 to 47 per cent of the group gaining in attainment. Thus, to demonstrate an effect of training in descriptive geometry we must do more than discover how many individuals gained. We must next examine carefully the amount of gain made by the gainers of each group. Turn to Table J-1 for a survey of the aggregate and average gains in "Attempts" and "Rights." The figures represent for 1913 and 1914 training and control groups the total and average number of problems (or parts of problems, as in Tests 1 and 2) gained in Tests 1 to 5, respectively. The data for the number of operations or problems gained in "Attempts" are as follows:

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Training Group.........	5.85	5.32	2.077	1.445	2.0
Control Group.........	5.56	4.34	1.705	1.582	1.615
Gain................	0.29	0.98	0.372	—0.137	0.385

This shows a residual gain in favor of the training group in each test except No. 4. In Test 1 the training

group gained 5.7 per cent more than the control group; in Test 2, 22.6 per cent more; in Test 3, 21.8 per cent more, and in Test 5, 24.2 per cent more. In Test 4 the figures given indicate that the control group gained 10 per cent more than the training group. We recall that here, as above in the discussion of efficiency in the entire groups, these figures do not take account of the individuals who solved all four problems of this test in less than allowed time. Of these there were only 22 out of 87 in the control group, 24.9 per cent; in the training group there were 133 out of 297, 44.8 per cent. No device for accurately measuring the amount of gain due to lower time has been worked out for this test. We do know, however, that, considering these figures, the gain of the training group considerably exceeds that of the control group.

The data for the average number of operations of problems gained in "Rights" are as follows:

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Training Group........	5.52	4.71	2.47	1.64	2.19
Control Group........	5.96	3.71	2.03	1.52	1.50
Gain...............	—0.44	1.00	0.44	0.12	0.69

With the exception of Test 1, the residual gains are again in favor of the training group. In Test 2 the training group gained 21.2 per cent more than the control group; in Test 3, 22 per cent more; in Test 4, 7.9 per cent more, and in Test 5, 46 per cent more.

Let us recall the data from Test 6 as found in Table D, in order to check our evidence on the gain in ability in non-geometrical manipulation. The number of words obtained by visual imagery alone gave a residual gain in favor of the training group of 7.4 per cent. It was also found that the total training group, as compared with the total control group, showed an increasingly greater supe-

riority as one proceeded from the non-geometrical to the strictly geometrical fields of mental maniputation. The figures were, for these three types of tests, roughly 7, 20, and 31 per cent.

In general, therefore, the data of the present inquiry into the amount of gain, agree with the previous results, in that the training group shows a considerably larger residual gain over the control group with strictly geometrical material than with non-geometrical material.

(c) *In How Many Tests Do the Gainers Gain?*

There still remains one method of attacking the problems of "gain" on the part of the various groups. Having determined the number of gainers and the proportion of all individuals gaining, and the relative amounts of the gains, we should now establish the frequency of occurrence of these gains. Do the same individuals gain in all of the tests? What is the typical number of tests in which members of the two groups gain?

In Table I is found a statement of the number of individuals who gained in various proportions of all tests taken. These totals are so arranged in the table that the columns from left to right progressively represent a constantly decreasing efficiency. For example, the totals for the training group read: 24 subjects worked 5 tests and gained in 5; 7 worked 4 and gained in 4; 4 worked 3 and gained in 3, etc.; 68 worked 5 and gained in 4; 19 worked 4 and gained in 3; 1 worked 3 and gained in 2; 98 worked 5 and gained in 3, etc. In addition to these detailed data, the summaries of the tables give the number gaining in 60 per cent or more of the tests taken, the number gaining in 20 per cent to 50 per cent of the tests taken and the number gaining in none of the tests taken. Further, to

typify the efficiency of the entire groups, we can state the average number of tests in which gains were made in "Attempts" and in "Rights."

Examining the columns of the table we find that for the training group the balance of the table is thrown heavily to the left or more efficient end. For the control group, however, the "center of efficiency" moves distinctly to the right of the middle, toward the less efficient end. Thus, even if we did not have at our command specific averages of gain made, we should be convinced of the greater constancy of gain on the part of the training group by the pictorial arrangement. To be more precise, however, how far does this greater constancy extend? In "Attempts" 67.8 per cent of the training group gained in 60 per cent or more of all tests taken; 42.5 per cent of the control group gained in 60 per cent of all tests taken—a further proof of the superiority of the training group in the amount gained. And, just as has been the case with other phases of the tabulations, this superiority is even more marked in "Rights" than in "Attempts." Seventy-two and seven-tenths per cent of the training group gained in 60 per cent or more of the tests taken, while 31.0 per cent of the control group gained in 60 per cent or more of the tests taken. But, going still further into the problem, what is the "psychological center of efficiency" for the table? What is the average number of tests in which gain is made by each of the groups? In "Attempts": for the training group $\frac{37.7}{60}$, or 63 per cent; for the control group $\frac{29.3}{60}$, or 49 per cent. In "Rights": for the training group $\frac{38.6}{60}$, or 64.5 per cent; for the control group $\frac{24.0}{60}$,

or 40 per cent. Thus, from all angles from which we may view the progress of the various groups it is found that the training group gains in a distinctly larger proportion of the tests than does the control group.

(*d*) *Summary.*

What, then, are the essential features of the perspective which has been drawn of the efficiency and progress of the subjects in the February and June test-series?

(*1*) *Efficiency of the entire groups.* (*a*) In initial attainment the training group slightly excels the control group in the mental manipulation of non-geometrical and quasi-geometrical elements and greatly excels the control group in the case of strictly geometrical elements. That is, the initial attainment of the training group is always superior to that of the control group. (*b*) Regardless of the status of initial attainment, the median gain in attainment of the entire training group exceeds that of the entire control group. The residual gains in favor of the training group are progressively greater as one proceeds from the non-geometrical field (7 per cent in Tests 1, 2, and 6), to the quasi-geometrical field (20 per cent in Test 3), and to the strictly geometrical field (31 per cent in Tests 4 and 5).

(*2*) *Increase in individual efficiency.* (*a*) How many individuals gain? In "Attempts," in all of the tests approximately half again as large a proportion of the training group gain as of the control group. In "Rights," considerably over half again as large a proportion of the training group gain as of the control group. (*b*) How much do they gain? In both "Attempts" and "Rights" the training group gains approximately 20 per cent more than the control group and makes a progressively increasing gain over the control group as one proceeds from the tests with

non-geometrical elements to those with quasi- and strictly geometrical elements. (*c*) In how many tests do the gainers gain? In "Attempts" 67.8 per cent of the training group and 42.5 per cent of the control group gain in 60 per cent or more of the tests taken. In "Rights" 72.7 per cent of the training group and 31 per cent of the control group gain in 60 per cent or more of the tests taken. The average per cent of tests in which the two groups gain is, for the training group, "Attempts" 63 per cent, "Rights" 64.5 per cent; for the control group, "Attempts" 49 per cent, "Rights" 40 per cent. In general, the training group gains in a distinctly larger proportion of tests taken than does the control group.

3. *The Correlation of the Mental Abilities Involved in Tests and Training Series.*

We now come to the fifth step in the statistical presentation of the results of this study, the discussion of the way in which the mental abilities involved in test and training series are related. In Tables K, L, and M are found correlations built up by the Pearson product-moment method that show the relationship between efficiency in the mental abilities measured by each of the various tests. They present fairly definite pictures of the way in which the abilities of the subjects of this investigation are related in the various types of mental manipulation. Granted that the tests measure efficiency in various types of mental manipulation, then these correlation tables give a further method of showing the relative effects of the training course upon our subjects.* In Table VI

*The author offers this discussion of "correlation" merely as further confirmatory evidence of the "transfer of training" in mental manipulation of spatial elements. Taken by themselves, the data that follow could not, of course, establish conclusively the possibility of transfer.

is given a summary of the values of *r*, the coefficient of correlation, for each of the tests with each of the others (both in February and in June) and of certain of the tests with the class marks in descriptive geometry. Corresponding unreliabilities (probable errors) are given. Certain correlations are also given for corresponding tests for the control group in order to establish the fact that increase in June correlations over February correlations was not due to practice-effect in taking the tests. The author has adapted to his problem the method of defining and interpreting the correlation coefficient described by Professor H. L. Rietz.*

Two measurable mental abilities may be said to be "correlated", if to any assigned values of one there are corresponding values of the other, whose *mean values* are *functions* of the selected values. "Function" is used here in the sense of a "mathematical function". In this sense the variable ability, *e. g.*, the mental manipulation of one type of spatial elements would be said to be a mathematical function of the ability to manipulate *another type*, if to any assigned degrees of efficiency in one there corresponded definite degrees of efficiency in the other. Correlation between two abilities suggests causal connections of greater or less degrees. The existence of correlation is most commonly represented by means of a single coefficient, called the Pearson coefficient of correlation, *r*. The degree of relationship between abilities is commonly inferred from the relative size of this coefficient. Thus, correlation may be spoken of as high, marked, low, zero, etc. It can be seen that the psychological interpretation of the numerical coefficient depends upon the arbitrary placing of the limits of the groups called 'high', 'low', 'marked', etc. The author's practice is to regard correlation as 'negligible' or 'indifferent' when *r* is less than .15 or .20; as being present, but 'low', when *r* ranges from .15 or .20 to .35 or .40; as being 'marked' from .40 to .50 or .60, and as being 'high' with values of *r* above .50 or .60. Again, it should be said that relationships worked by the product-moment method should be interpreted with careful regard for the 'spread' of the table itself, not from the size of the coefficient alone. It should also be emphasized that the product-moment method is valid for determining the degree of relationship between two quantities when the mean positions of successive arrays of the contingency table fall approximately on a straight line. The checking up of the linearity of this line of the

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means of arrays (known from Galton's terminology as the "line of regression") is absolutely necessary to the valid use of Pearson's product-moment method. The linearity of regression in each of the correlation tables drawn up in this study has been checked and the Pearson formula shown to be applicable. Much statistical work in educational research has been open to the criticism that the applicability of methods used has not been adequately determined.

TABLE VI.

DETERMINED VALUES OF THE PEARSON COEFFICIENT *r* FOR VARIOUS TESTS IN MENTAL MANIPULATION.

Test Results Correlated	Training Group February Tests	Training Group June Tests	Control Group February Tests	Control Group June Tests
1 and 2	.61 ± .018	.69 ± .021	....	....
1 and 3	.24 ± .034	.44 ± .033	.20 ± .072	.18 ± .073
1 and 4	.13 ± .038	.19 ± .038	....	....
1 and 5	.20 ± .038	.26 ± .037	.24 ± .05	.16 ± .05
2 and 3	.11 ± .039	.24 ± .039	.19 ± .05	.19 ± .05
2 and 4	.16 ± .038	.27 ± .036	....	....
2 and 5	.25 ± .036	.20 ± .038	....	....
3 and 4	.22 ± .037	.23 ± .039	....	....
3 and 5	.28 ± .033	.37 ± .035	.18 ± .06	.17 ± .06
4 and 5	.26 ± .036	.42 ± .032	.42 ± .05	.48 ± .05
1 and 6 (v)*	.33 ± .036	.31 ± .036	....	....
1 and 6 (v and a)	.32 ± .034	.36 ± .036	....	....
D. G. and 2	.26 ± .037	....	....	....
D. G. and 5	.22 ± .036	.41 ± .032	....	....

*'*v*' in the correlation of 1 and 6 refers to the data for number of words obtained by visual imagery alone; '*v and a*' refers to words obtained by visual and auditory methods together.

Analysis of these correlation tables and of the values of *r* given in Table VI will throw light on certain fundamental issues in our experimental field. The problem may be approached by stating certain essential premises and then checking these by the statistical results:

First, if a specific mental ability is measured by two mental tests (assuming that the tests are adequate measures of the ability in question), the correlation of efficiency in the two tests should be high, approaching 1.00.

Second, if two mental abilities are largely unlike, *i. e.,* if they comprise many unlike elements, then the correlation of efficiency in the two abilities, as measured by the same mental test, should be a small, positive one or a negative one, and should become the less positive as the abilities comprise progressively fewer related elements or elementary processes.

Third, if two such unlike mental abilities are influenced by a training series so as to affect the same proportional increase in the efficiency of both mental abilities, then the correlation between the resultant efficiencies in the two abilities should remain constant.

Fourth, if two such unlike mental abilities are influenced positively by a training series *in unlike degree* in such a way as to increase the efficiency in one more rapidly than in the other, then the correlation in efficiency should vary, becoming increasingly higher if the training series causes efficiency in one ability to approach more closely the efficiency in the other ability.

A survey of the correlation tables and of the values of r leads to the following conclusions:

First, the high correlations between Tests 1 and 2 indicate the presence of the same mental ability or of mental abilities that comprise very nearly similar elements.

Second, the correlations obtained for the control subjects indicate that an increase in the amount of r in the case of the training subjects cannot be ascribed to the practice-effects in the tests themselves.

Third, the correlations between either Tests 1 or 2 and 3, 4, or 5 indicate the presence of dissimilar abilities, as the correlations are low—on the average less than .20.

Fourth, the non-geometrical abilities measured by Tests 1 and 2 have been influenced by the training-series positively (as witness the per cent gains indicated in the pre-

ceding sections) and in such manner as to cause their efficiency to approach more closely to that of the geometrical mental abilities measured by Tests 3, 4, and 5.

Fifth, the correlations for Tests 1, 2, and 6 indicate that Tests 1, 2, and 6 do not measure the same mental abilities. They confirm the statements made in the discussion of the method of scoring the tests, viz., that the solution of Test 6 involves the activity of several elementary abilities, some of which are not specifically affected by the training course. Were this not true, the correlations of the non-geometrical tests should be high, and the correlations of the June tests higher than those given in January.

Sixth, Tests 3, 4, and 5, although designed as measures of so-called 'geometrical' manipulation, prove to be measures of abilities dealing with many unlike elements, for the correlations average 0.253. Thus, these mental abilities previously described as strictly or quasi-geometrical mental abilities are probably complexes of simpler mental abilities. The figures indicate that these abilities do belong to more closely related fields of mental manipulation than do all five tests when taken together. The average of the correlations of Tests 3, 4, and 5 with each other is 0.34, while the average of the correlations of Tests 1 and 2 with 3, 4, and 5 is 0.27.

The laborious tabulation and computation of these values of *r* at least offers evidence confirmatory of our general thesis. It brings out the complexity of the interaction of mental abilities in such seemingly simple and related fields as the mental manipulation of spatial elements of geometrical types in a way that a discussion of absolute efficiency of the training and control groups, relative gains, number of gainers, etc., cannot. At the same time, it does lend support to the conclusion that something in the training received by the training subjects

raised the efficiency in non-geometrical manipulation, and this in spite of the fact that the training as given dealt only with geometrical elements.

4. *The Source of Residual Gains in Favor of the Training Group.*

Our statistics have established at least three important facts: (1) the training group contains a larger proportion of gainers than the control group; (2) the average gain of the gainers in the training group considerably exceeds the average of the gainers of the control group; (3) the training group gains in a distinctly larger proportion of the tests taken than does the control group.

During the four months of training something in the mental activities of the training group caused a considerably greater increase of efficiency in the mental manipulation of spatial elements than was attained by our control group. Recalling the conditions of this investigation as stated in our description of the problem, we are forced to the conclusion that some one or more of the subjects of study pursued by the training group during the interval contributed the training necessary to increase this efficiency. To what subject or subjects is this residual efficiency due?

(*a*) *Discussion of the Subject-Matter of Courses Studied by the Training Group.*

In choosing a problem for classroom investigation of transfer we deliberately selected the manipulation of spatial elements as the ability to be trained and descriptive geometry as the subject of study to train it, because of the constancy with which mental manipulation was con-

sciously made a daily factor in the solution of problems. The author was convinced by his experience in teaching various subjects of the civil-engineering curriculum and by an intensive study of the other engineering curriculums that there is no other course of study offered to first and second-year students that demands a daily conscious use of visualization. Let us first satisfy ourselves of the truth of this assertion by an examination of the subject-matter of the other courses pursued by our training group during the period of training. The courses of study in the engineering curriculums of the University of Illinois in their first two years are made up of prescribed subjects. Irregularities in our groups are therefore few, and only 12 different semester-courses are represented in the list of elections and prescriptions taken by as many as 10 of the training subjects. Descriptive geometry is the one subject studied by each of our subjects. Algebra, trigonometry and analytic geometry make up the principal requirements in mathematics, and most of the training subjects take the last-named course. Of 203 individuals who take mathematics, 177, or 87.4 per cent, of them take analytic geometry. That the functioning of "mental manipulation of spatial elements" in the present teaching of college algebra is very slight and may be neglected for the purposes of this investigation will undoubtedly be agreed. In analytic geometry there may be incidental instances in which visual factors reinforce the analytic method. However, the entire subject is, as its name indicates, of an analytical rather than a descriptive nature, and any training in visualization received from a semester's study of it must be purely incidental and negligible.

The chief distinction in grouping concerns the language requirements—English, rhetoric, French or German. In

grouping the subjects into the seven training sub-sections the most frequently elected subjects, in common with descriptive geometry and shop-practice, were found to be either a modern language or English. Search for the possible training in mental manipulation that might come from an elementary course in French, German or Spanish indicates that it could function, if at all, only in connection with the mastery of vocabulary (memory) and then the mental manipulation would probably be largely of the auditory or motor rather than of the visual type. Further, it can be conceived that constant daily emphasis by the instructor on the necessity of exercising the imagination in the writing of themes (English) might later be effective in increasing efficiency in mental manipulation of a non-geometrical type. If so, our statisctics on efficiency of training sub-section No. 1 should show a larger gain in Tests 1, 2, and 3. This does not prove to be the case, as will be shown later. No such daily emphasis is given by the average instructor in the teaching of English composition or of rhetoric to freshmen engineers.

Shop-practice and architecture remain as the only other courses studied by considerable numbers of the training subjects. It is believed that we may neglect the training qualities produced in mental manipulation of spatial elements by a semester's work in the machine or forge shop and likewise by the purely historical study of architecture. The practice in "design" in architecture might admittedly contain considerable incidental emphasis on visualization. If so, it should result in increased attainments in mental manipulation on the part of the training subjects electing it for study. Especially should this be true in the geometrical fields. That it does not do so in the case of the few men pursuing studies in this field will be brought out a little later in statistics on the efficiency

of the training sub-sections. Similarly with the small number of individuals studying laboratory and theoretical physics and chemistry; here the continued use of apparatus probably tends to discourage mental manipulation. However, even should we grant the possibility of visual training coming from the study of chemistry, physics and architecture, the small number of individuals that pursued these studies could not materially affect the statistical results.

In brief, then, long study of the ingredients of the various engineering curriculums and actual teaching experience in at least nine of the courses of the civil engineering curriculum convinces me that descriptive geometry is the only subject of study pursued by the members of the training group that consciously and explicitly offers daily training in the mental manipulation of spatial elements as an accepted device for the solution of problems, and that the training effect for mental manipulation of all the other courses studied by the training subjects may be considered incidental, intermittent, and negligible for the purposes of this investigation.

(*b*) *Statistical Analysis of the Problem.*

Added strength is given to the foregoing by a statistical treatment of the problem. In Table J the data for the training subjects are grouped in seven sub-sections, as indicated in the chapter on "The Subjects." In this table is tabulated, for each training sub-section and for each test, 1 to 5 inclusive, the number of individuals gaining and the per cent of the total number of individuals gaining, the average gain of those gaining and the deviations of the mean of each training sub-section from the means of the whole group. Now, if the courses pursued by any

one section exert a greater training effect than those of any other section, this should be evidenced (in the type of visual manipulation trained) by a higher percentage of individuals gaining in the tests and by a higher average gain. A brief survey of the statistical results for each of these sub-sections will now be made.

Group 1 (55 subjects). English is the particular course under analysis. If a semester's study in English composition or rhetoric fulfills a function in training abilities in the mental manipulation of non-geometrical elements, the per cent of individuals in Group 1 gaining in Tests 1 and 2 should exceed the per cent gaining in the whole group, and their average gain should also be higher. Table J tells us, however, that the averages for Group 1, instead of being higher than the averages of the entire training group in these two tests, are actually very slightly lower. The per cent of individuals gaining in Group 1 and the entire training group and the deviations of the means of per cent gains of Group 1 from the means of the entire training group are abstracted in Table VII.

TABLE VII.

COMPARISON OF GROUP 1 WITH ENTIRE TRAINING GROUP.

A. Per Cent. Who Gain.

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Group 1	82.0	62.0	57.5	64.0	73.0
Training Group	74.0	67.8	57.6	55.3	62.5
Difference	8.0	—4.8	—.1	8.7	10.5

B. Difference in Percentual Amount of Gain.

—4.4	—0.4	26.8	4.2	—8.4

We may say, then, that the results of the tests indicate that English courses add nothing to the increased efficiency of the subjects in mental manipulation of non-geometrical elements.

Group 2. French and German are the particular courses under investigation. Practically the same situation exists here as with Group 1 (119 individuals pursue study in modern languages in addition to descriptive geometry, shop-practice and analytic geometry). Here, again, we are especially interested in the results of Tests 1 and 2, as modern languages would affect non-geometrical elements if any part of the field of mental manipulation. The figures for Group 2 appear in Table VIII:

TABLE VIII.

COMPARISON OF GROUP 2 WITH ENTIRE TRAINING GROUP.

A. Per Cent. Who Gain.

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Group 2	69.0	60.5	60.7	55.3	62.7
Training Group	74.0	67.8	57.6	55.3	62.5
Difference	—5.0	—7.3	2.9	0.0	.2

B. Difference in Percentual Amount of Gain.

3.7	6.5	—8.5	1.9	9.8

The figures show that fewer subjects in Group 2 gained in Tests 1 and 2 than in the whole training group, and that almost the same number gained in Tests 3, 4, and 5, as in the whole group. There is clearly little evidence here for greater training with non-geometrical elements. The differences in amount gained by Group 2 are slightly positive in all tests but Test 3, but they are small throughout, and in the case of Tests 1 and 2 are so small as to have no

great weight, for they are smaller than the differences for Test 5, a strictly geometrical test. Thus, the statistical data for the two largest and most important subsections do not prove the existence of training in visualization in any subject of study except that taken by all our trained subjects, descriptive geometry.

Group 3. Since those subjects electing science (physics and chemistry) are so few in number (8 individuals), it is hardly necessary to discuss their abstracted data in detail. Suffice it to say that the gains for the group are neither consistently positive nor negative, but distribute themselves on both sides of the means of the training group.

Groups 4 and 5. These groups are made up of 20 subjects electing study in architecture, descriptive geometry and analytic geometry. The data for this group appear in Table IX.

TABLE IX.

COMPARISON OF GROUPS 4 AND 5 WITH ENTIRE TRAINING GROUP.

A. Per Cent. Who Gain.

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Groups 4 and 5.......	65.0	85.0	32.0	50.0	65.0
Training Group.......	74.0	67.8	57.6	55.3	62.5
Difference	—9.0	17.8	—25.6	—5.3	2.5

B. Difference in Percentual Amount of Gain.

—16.8	—12.2	—55.4	—23.5	—22.7

What can we infer from these figures? First, in the non-geometrical tests, averaging the results for Tests 1 and 2, about as many subjects gain in Groups 4 and 5 as in the entire training group; in partially and strictly

geometrical tests fewer subjects gain in Groups 4 and 5 than in the entire training group. Second, the average gain of Groups 4 and 5 is in all tests very much lower than the average gain of the entire training group. At least, it is certain that the courses pursued by these groups have not produced efficiency greater than that of the entire training group.

Group 6. The 67 subjects of this group were registered in advanced courses in science and language, and had previously completed the requirements in shop-practice. If such material for study has offered training in mental manipulation of spatial elements, the figures should indicate its presence in increased efficiency. The data for Group 6 appear in Table X.

TABLE X.

COMPARISON OF GROUP 6 WITH ENTIRE TRAINING GROUP.

A. Per Cent. Who Gain.

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Group 6	83.0	58.0	55.6	54.5	60.7
Training Group	74.0	67.8	57.6	55.3	62.5
Difference	9.0	—9.8	—2.0	—0.8	1.8

B. Difference in Percentual Amount of Gain.

—6.7	1.7	20.2	—18.1	31.8

With a group of individuals having a possible advantage in substituting for shop-practice advanced courses which might be thought to give added training in mental manipulation, the situation is practically the same as in the other training sub-sections. There is no consistently larger gain or larger number of individuals gaining with either non-geometrical or geometrical elements.

Group 7 is a group of 37 subjects, mostly special students who take two mathematics courses, two or more English courses, or two or more language courses, and also all those who take advanced courses, *e. g.,* mechanics, etc. The data appear in Table XI.

TABLE XI.

COMPARISON OF GROUP 7 WITH ENTIRE TRAINING GROUP.

A. Per Cent. Who Gain.

	Test 1.	Test 2.	Test 3.	Test 4.	Test 5.
Group 7	79.5	83.0	56.0	58.5	59.6
Training Group	74.0	67.8	57.6	55.3	62.5
Difference	5.5	15.2	—0.4	3.2	—2.9

B. Difference in Percentual Amount of Gain.

5.3	—3.8	—29.8	24.0	—18.4

With the exception of Test 2, the proportion of individuals gaining in Group 7 is practically identical with the proportion gaining in the whole training group, and the predominance of negative differences in average gains of the group does not indicate that the advanced courses have given distinct training in mental manipulation.

(*c*) *Summary of the Source of Residual Gains.*

From the analytical discussion of the cause of greater increase in efficiency on the part of the training group the conclusion is reached that of all the courses of study pursued by the subjects during the period of training, descriptive geometry—which was selected as the training course because of its constant emphasis on the mental manipulation of geometrical spatial elements—was the only course whose subject-matter, organization, and meth-

ods of teaching could so contribute training in mental manipulation as to function in the results of the tests. From the statistical analysis of the actual performance of subjects who pursued different courses of study the conclusion is reached that the statistical evidence does not indicate that training actually found could have come from any course except the one designed and selected expressly for the purpose and studied by all training subjects of the investigation—descriptive geometry.

5. *The Effect of Training on Different Grades of Scholastic Ability.*

The large number of subjects used in this investigation makes it possible to contribute evidence on several other points of issue in the field of transfer. The first of these is: "What effect does a given training series have upon different grades of scholastic ability?"

It was assumed that scholastic ability is approximately measured by the marks received in college courses. The marks received by each training subject were collected and are tabulated in Tables A-1, A-2, A-3. The courses included mathematics, English, modern languages (French, German and Spanish), shop-practice, mechanical drawing and descriptive geometry. The mathematics mark is, in each case, an average of two or more semester marks in freshman algebra, trigonometry and analytic geometry; the English mark, an average of two semester marks in theme-writing and English composition; the modern language mark, an average of two semester marks in French, German or Spanish; the marks in drawing, shop-practice and descriptive geometry each represent the standing in one semester course. For each of the tests 1, 2, 3, and 5 the 1913 training subjects were next grouped

according to initial performance. Each group contained only those who had obtained approximately the same initial efficiency in any one test. For example, all subjects were grouped together whose initial efficiency in Test 1 (as shown by the scoring of the February test-series) was between 20 and 30 per cent, between 30 and 47 per cent, etc. The use of this method results in groups whose gains are comparable.

The gains in each test were handled separately; detailed tabulations were made and the individual results were distributed under "Gained" or "Not Gained" columns, according to the mark received in the various courses of study. From these original tables, Summary-Table XII was drawn up to show for the two groups of students (the one with marks below 75 per cent and the other with marks between 85 and 100 per cent) the number gaining or not gaining, the per cent gaining or not gaining and the average gain. The data presented in Table XII are grouped in accordance with marks obtained in mathematics, in descriptive geometry and in English and modern languages.

With regard to marks in mathematics, the data show that a larger proportion of those above 85 than of those below 75 gained in 7 out of 10 groupings; the residual gains in their favor are for the tests, respectively, 20, 30, 48, 2, 18.8, 13, and 32 per cent. Those below 75 per cent in mathematics show a larger per cent of their number gaining in only 3 out of 10 groupings; the residual gains in their favor are 13, 4, and 9 per cent. The average gain in each test for students above 85 per cent, compared with students below 75 per cent, gives almost identical results in all but one group.

With regard to marks in English and modern languages, in 4 out of 6 groupings a larger proportion of the students above 85 per cent gained than of the students below 75 per cent; the residual gains in favor of the higher group were 4, 13, 14, and 4 per cent. In 2 out of 6 groupings a larger proportion of the students below 75 per cent gained than of the students above 85 per cent; the residual per cents of the number gaining were 7 per cent and 4 per cent. Here, again, the amount of gain was with each group approximately the same.

With these representative groupings of the training subjects, comparable results in each case permit the following conclusions: (1) a considerably larger proportion of the students who showed high ability in *mathematics* gained in ability in the mental manipulation of spatial elements than of the students who showed low ability in mathematics; (2) in those groupings in which there was a larger proportion of the poor students excelling the good students, the differences on the whole were very slight; (3) a comparison of the effect of training on good and poor students in *English and modern languages* shows but a very slight advantage for the good students; the residuals in all groupings are very small.

It has been suggested that these statistical results may throw light upon the method of transfer in this investigation. Those who attribute transfer to the development of concepts or ideals of method may find here some support for their position. Mathematics courses, like algebra, trigonometry, analytic geometry and descriptive geometry, are primarily "problem" courses; the best success in them demands the development of methods of attack or analysis which require a conceptualizing ability of a relatively high order. This is most clearly brought out in the presentation of the course in descriptive geometry. In the working

TABLE XII.

NUMBER AND PER CENT OF TRAINING SUBJECTS GAINING AND THE AVERAGE GAINS, DISTRIBUTED ACCORDING TO GRADES RECEIVED IN MATHEMATICS, DESCRIPTIVE GEOMETRY, AND ENGLISH AND MODERN LANGUAGES.

A. Mathematics.

Grade Received.	Number Gaining.	Number Losing.	Per cent Gaining.	Per cent Losing.	Average Gain in Per cent.
Test 1. Initial Efficiency 30 to 47 Per Cent.					
85-100.............	25	2	93	7	45.0
Below 75..........	24	9	73	27	47.8
Test 1. Initial Efficiency 50 to 60 Per Cent.					
85-100.............	12	2	86	14	....
Below 75..........	9	7	56	44	....
Test 1. Initial Efficiency Above 60 Per Cent.					
85-100.............	9	5	64	36	....
Below 75..........	1	5	16	84	....
Test 2. Initial Efficiency 20 to 40 Per Cent.					
85-100.............	15	5	75	25	46.4
Below 75..........	29	7	88	12	42.3
Test 2. Initial Efficiency 41 to 60 Per Cent.					
85-100.............	21	17	55	45	14.6
Below 75..........	13	9	59	41	19.9
Test 3. Initial Efficiency 50 to 67 Per Cent.					
85-100.............	15	22	41	59	....
Below 75..........	9	16	39	61	....
Test 3. Initial Efficiency 25 to 42 Per Cent.					
85-100.............	12	8	60	40	....
Below 75..........	9	4	69	31	....
Test 5. Initial Efficiency 50 Per Cent. or More.					
85-100.............	11	6	65	35	....
Below 75..........	5	10	33	67	....

B. Descriptive Geometry.

Test 1. Initial Efficiency 30 to 47 Per Cent.

Grade Received.	Number Gaining.	Number Losing.	Per cent Gaining.	Per cent Losing.	Average Gain in Per cent.
85-100.............	34	3	92	8	40.8
Below 75..........	19	7	73	27	44.4

Test 2. Initial Efficiency 20 to 40 Per Cent.

85-100.............	19	4	83	17	56.4
Below 75..........	24	10	70	30	34.4

C. English or Modern Languages.

Test 1. Initial Efficiency 30 to 47 Per Cent.

Grade Received.	Number Gaining.	Number Losing.	Per cent Gaining.	Per cent Losing.	Average Gain in Per cent.
85-100.............	18	3	86	14	44.4
Below 75..........	28	6	82	18	49.0

Test 2. Initial Efficiency 20 to 40 Per Cent.

85-100.............	8	4	67	33	49.3
Below 75..........	26	9	74	26	47.6

Test 2. Initial Efficiency 41 to 60 Per Cent.

85-100.............	22	13	63	37	13.0
Below 75..........	14	14	50	50	22.5

Test 3. Initial Efficiency 25 to 42 Per Cent.

85-100.............	11	8	58	42	....
Below 75..........	13	8	62	38	....

Test 3. Initial Efficiency 50 to 67 Per Cent.

85-100.............	12	14	46	54	....
Below 75..........	6	13	32	68	....

Test 5. Initial Efficiency 50 Per Cent. or More.

85-100.............	7	9	44	56	....
Below 75..........	10	15	40	60	....

of every problem analysis is stressed, because otherwise the solution of almost none of the problems is possible. The author in his own classes considered the development of a "general method" of problem solution to be one of the fundamental purposes of the course, and tried to make it one of the definite outcomes of his teaching. On the other hand, it is improbable that conceptualizing abilities play so large a rôle in determining excellence in languages. Thus, our argument, since it is to some extent supported by our statistical results, leads to the inference that in this investigation the method of transfer involves, among other things, the conceptualizing of methods of attacking problems. Since the training seems to be more effective with the high-grade students of mathematics than with the low-grade students of mathematics, and to have practically identical effects on students of various grades of ability in the languages, it may very well be that the training 'carries over' through this very conceptualizing process. Naturally, there is in this no positive proof that concepts of method constitute the only agency of transfer.

6. *The Effect of Training Upon Subjects of Various Ages.*

To test the effect of training upon different levels of maturity the writer tabulated (in Tables A-1, A-2, A-3) the ages of each of the training subjects at the time of taking the tests. Distribution tables were drawn up for each of the tests 1, 2, 3, 4, and 5, showing the number gaining and the number not gaining for each half-yearly interval from 16 years to 26 years. Table XIII summarizes these facts.

If we draw a dividing line at 20 years, the figures summarize as follows: of the subjects 17 to 20 years old, 280 gain, 185 do not gain; or, in per cents, 60.2 gain and 39.8 do not gain. Of the subjects 20 to 26 years old, 252 gain

TABLE XIII.

NUMBER OF TRAINING SUBJECTS WHO GAIN OR DO NOT GAIN, DISTRIBUTED ACCORDING TO AGES BY HALF-YEARLY INTERVALS.

Age (in Years and Months).

Test	17:1—17:6		17:7—18		18:1—18:6		18:7—19		19:1—19:6		19:7—20		20:1—20:6		20:7—21		21:1—21:6	
No.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.
1........	4	0	5	0	7	4	23	9	11	4	22	10	26	3	13	6	5	1
2........	2	2	3	2	5	3	20	9	9	6	20	10	18	8	9	8	2	3
3........	2	1	4	2	8	3	19	13	9	7	12	19	17	10	10	9	5	1
4........	2	2	2	3	3	4	12	14	6	8	17	14	17	11	13	5	3	3
5........	1	3	4	1	5	2	14	14	10	5	19	11	17	11	10	8	3	3
Total..	11	8	18	8	28	16	88	59	45	30	90	64	95	43	55	36	18	11

Test	21:7—22		22:1—22:6		22:7—23		23:1—23:6		23:7—24		24:1—24:6		24:7—25		25:1—25:6		25:7—26	
No.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.	G.	N. G.
1........	6	3	5	2	5	0	0	1	3	0	2	1	..	..	1	0	2	0
2........	3	3	5	2	3	2	0	1	3	0	1	1	..	..	1	0	1	1
3........	5	4	3	4	1	4	1	0	1	2	2	1	..	..	0	1	0	2
4........	1	6	2	5	4	2	0	1	3	0	1	2	..	..	0	1	2	0
5........	6	1	3	3	2	3	0	1	3	0	2	1	..	..	1	0	0	2
Total..	21	17	18	16	15	11	1	4	13	2	8	6	..	..	3	2	5	5

and 153 do not gain; or, in per cents, 62.5 gain and 37.5 do not gain. Comparing the subjects younger than 18 years with those older than 22 years, we find 64.4 per cent of those younger than 18 years gaining and 57.8 per cent of those older than 22 years gaining. It seems then, that the training given in the mental manipulation of spatial elements affects the abilities of the younger and the older adult subjects of this investigation to approximately the same degree.

7. *The Correlation of Scholastic Ability in Various College Studies.*

The large number of subjects in the training group enables us to present still other kinds of data often discussed in connection with the transfer problem—particularly the correlation of ability in various scholastic studies. To supply these data the marks received in mathematics, English or modern languages, descriptive geometry, shop-practice and mechanical drawing were tabulated in Tables A-1, A-2, A-3, and retabulated in correlation tables M-1 to M-8. From these tables the Pearson coefficient of correlation, r, was next computed to show the relationship between scholastic ability in mathematics and in descriptive geometry, between ability in mathematics and in modern languages, between mathematics and English, mathematics and shop-practice, mechanical drawing and shop-practice. These values of r are given in Table XIV.

The results are confirmatory of the results of a similar but more extended study in correlation of scholastic abilities by Rietz and Shade,* conducted at the University of

*Rietz and Shade: "Correlation of Efficiency in Mathematics and Efficiency in Other Subjects." *The University of Illinois Studies*, VI, No. 10 (1908).

TABLE XIV.

CORRELATION OF SCHOLASTIC ABILITY IN VARIOUS COLLEGE STUDIES.

Subjects Correlated.	Value of r.
Mathematics and Descriptive Geometry	.70
Mathematics and Modern Languages	.50
Mathematics and English	.40
Mathematics and Shop-Practice	.44
Mathematics and Shop-Practice	.38
Mechanical Drawing and Shop-Practice	.44

Illinois and published in 1907. The coefficients found in the two investigations between groups of studies that were previously asserted (by such investigators as Lewis of Dartmouth**) to be negatively correlated are here in all cases positive and are practically identical. It may be concluded that the scholastic abilities of the training subjects of this investigation, as measured by their college marks, are positively correlated to a rather marked degree. Students who show high degrees of ability in certain mathematical courses will show practically similar degrees of ability in other mathematical courses; students who grade high in mathematics will tend to grade high in English, modern languages and shop-practice; those high in mechanical drawing will tend to do work of a similar quality in shop-practice. Thus, the complex abilities exhibited in various subjects of study, as measured by teachers' marks,* are positively correlated, and in general to a degree that may be characterized as "marked" and in many cases as "high."

**For a discussion of certain fallacies in the conclusions reported by Lewis, see Rietz, *Jour. Educ. Psych.*, Feb., 1916.

*For a complete discussion of the literature of "Teachers' Marks and Marking Systems," see article in *Educational Administration and Supervision*, February, 1915.

CHAPTER X.

The Theory of Transfer.

1. Summary of Statistical Results.

What contributions can this study offer to the discussion of transfer of training? Do our results indicate that the training of one mental ability will produce increased efficiency in other mental abilities more or less closely related? Does the effect of training in the mental manipulation of spatial elements, obtained from the definite organization of a course of study in geometry, function to increase the efficiency of mental manipulation of spatial elements in geometrical fields apart from this definite organization of the subject-matter of the geometry course itself? Does it function in fields only partly geometrical or of a different geometrical content? Does it function in fields not at all geometrical?

In formulating answers to these questions we are attacking the core of the problem of transfer. The issue before us is not merely: "Is there a spread of improvement among mental abilities?" It has two other and more important aspects: first, What is the extent of transfer or spread of improvement? How far does the effect spread? Second, How was the spread of improvement affected? How does it happen that improvement in one mental ability, effective in one field of mental activity, is corre-

lated with improvement in other mental abilities effective in seemingly quite unrelated fields? Through what channels and by what processes does the spread of improvement take place?

Thus our general problem is threefold (1) Does improvement in one mental ability spread to other abilities? (2) If so, how far does it spread? (3) Through what agencies does it spread?

Chapters III to IX, inclusive, have dealt exclusively with the first two phases of the general problem. The problem has been stated as restricted to a definite field. The conditions of selecting subjects, designing tests, giving the tests, scoring the test-papers and tabulating results have been described in detail above. It must be borne in mind that the tests are regarded as measuring efficiency in the function studied; that the conditions under which they were taken are regarded as normal, and as not influencing the progress of the experiment in any deleterious way, and that the papers were accurately scored and tabulated.

What, then, has this study shown in regard to the spread of improvement in abilities in mental manipulation of spatial elements?

(*a*) A group of 413 college students received such training in the mental manipulation of geometrical spatial elements as was naturally given in the conduct of a course in descriptive geometry during a collegiate semester of 15 weeks. Another group of 87 college students received no such training during that interval. Both groups were measured for efficiency in mental manipulation of

spatial elements before they received training, and the trained group proved to be considerably superior to the untrained group in manipulating geometrical elements and slightly superior to the untrained group in manipulating non-geometrical and quasi-geometrical elements. After the 15 weeks of training the trained group had increased its superiority over the untrained group in all three types of problems, geometrical, quasi- and non-geometrical elements—this, in spite of the fact that the training had been concerned with problems involving strictly geometrical elements.

(*b*) Members of both the trained and untrained groups revealed increased efficiency in the problems of the second test-series. But there were 44 per cent more gainers in speed in the trained group than in the untrained group, *i. e.,* nearly half again as large a proportion of the trained group gained as of the untrained group. And nearly two-thirds again as large a proportion of the trained group as of the untrained group gained in accuracy.

(*c*) The residual gain of the trained group over that of the untrained group was increasingly greater as one passed from the non-geometrical abilities to those partaking more and more of geometrical qualities. That is, the students trained in geometrical manipulation gained more than did the untrained students in efficiency of the functions tested, but their gain was greater in those abilities that involved the geometrical elements.

(*d*) The gainers among the trained students gained in a distinctly larger proportion of the tests taken than did the gainers of the untrained group. Or, to put it in terms

more pertinent to the point at issue, *the trained students gained in efficiency in more kinds of mental manipulation of spatial elements than did the untrained students.*

Translated into terms of transfer, what do the above facts indicate? They indicate, at least to the author's satisfaction, that the training in mental manipulation of a geometrical character received by students in descriptive geometry operated (1) so as substantially to increase the student's ability in solving 'manipulation' problems of a geometrical nature, but entirely unrelated to the content of descriptive geometry itself, (2) so as to increase the student's ability in solving manipulation problems of a slightly geometrical nature, *i. e.*, those containing the fundamental elements of geometry (the point, line and plane), but in such form as not to offer the same sort of geometrical 'cues' to aid in solution, (3) so as to increase the student's ability in solving problems in mental manipulation of a purely non-geometrical nature, *i. e.*, those in which the point, line, and plane do not appear in any way whatsoever. The training operates more effectively in those problems whose visual 'setting' the more closely resembles the visual imagery involved in descriptive geometry.

2. *The Range Over Which Training is Effective.*

From our statistical analysis it would seem that the facts argue for the spread of the effect of training mental abilities. Consider our data from whatever angle we will, we face the fact that the training subjects are relatively more efficient after training than before. Practice-effect,

increased maturity, and familiarity with the tests have all been accounted for carefully by the record of the control group, and still there is a distinct residual increased efficiency in the abilities that were not specifically trained by the training course.

But having satisfied ourselves that the effect of training did spread to abilities not specifically trained by the training series, can we go further and offer any definite information as to the exact range of this spread of improvement? Can we say, for example, that with the average student the training carried over one-half or four-tenths as efficiently into quasi-geometrical fields as it did into fields dealing with strictly geometrical elements? Or that it carried over one-third or one-fourth as efficiently with non-geometrical elements as with strictly geometrical elements?

Obviously we cannot. In order to do so we should require a definite measure of the adequacy of each test as a measure of the specific abilities tested. If we knew, for example, that Tests 1, 2, and 6 measured mental manipulation of a non-geometrical character to the exact extent that Test 3 measured efficiency in mental manipulation of a quasi-geometrical character, and that Tests 4 and 5 measured strictly geometrical manipulation, then we should be justified in expressing our relative per cent gains in the various tests as ratios of the extent to which training had spread. To know that, however, would require the solution of a problem in the design of mental tests, and in the calibration of mental tests, which in itself would be very formidable, and one whose solution has not

been deemed possible as a preliminary step in the conduct of this study.

At best, we must confine ourselves to saying that the effect of training is not so evident in non-geometrical abilities as in mental abilities dealing with geometrical elements, but that at the same time there is unquestionable evidence of its presence even there. Careful evaluation of the tests used with the various kinds of elements has not yet resulted in the assurance that we can express the range in definite units of measurement. In future investigations of this nature the emphasis should be upon this phase of the problem, for it clearly is of primary importance.

The final question in this invetigation of the transfer of training is: How did the effect of training in mental manipulation of a strictly geometrical nature spread so as to be effective with non-geometrical problems? Through what agencies does the improvement of mental abilities of this sort spread? How does training in manipulating the spatial relations of points, lines, planes, and solids raise the level of efficiency in manipulating the numerals involved in problems of short division?

3. *The Agencies of Transfer: A Study of the Processes Involved in the Solution of Tests of Efficiency in Mental Manipulation and in the Solution of Descriptive Geometry Problems.*

Statistics can no longer aid in the solution of the problem. It must now be studied through the careful analysis

of all conscious processes involved in manipulating the elements of our test problems. We must observe and record as minutely as possible each step in the mental process of solving both the problems of the tests and the problems of descriptive geometry. We must secure introspections on these processes from trained observers. Ideally we should have accurate records of all steps that took place in the consciousness of the subjects as they solved each of the tests. As the subjects were untrained in introspective analysis, this step was, of course, impossible. We did secure from each of our subjects brief introspective data on the various tests, but scarcely any of them are definite enough to give the information desired. For that reason the tests were submitted to four students of education and psychology, all of whom had had training in introspection. They were also given to a small class in descriptive geometry in 1914, with careful directions and questions regarding introspection. Each test was solved under experimental conditions, and was immediately followed by the writing of a full description of the mental process involved. The subjects were then asked to answer, for each test, a list of questions which had been designed by the author after many hours of introspective analysis of the tests and of the solution of descriptive-geometry problems. In this way a few fairly reliable introspections have been secured to aid in the discussion of the important question of the method of transfer as it applies to this investigation.

To illustrate definitely the mental processes involved in the solution of the tests there are quoted below cer-

tain pertinent extracts from the introspections which bear on the problem of the method of improvement of mental abilities. The detailed introspections are much too voluminous and contain too much irrelevant matter for complete publication. Enough has been quoted to represent fairly the records obtained.

A. Quotations from Introspective Analyses of the Imagery Processes Involved in the Tests.

Test 1. "When instructions were being given, felt that mind was receptive; followed easily with some little interruption of objects on desk and thoughts until 'divide 84 by 17' came. Then I began to try and divide, and became panicky when I could not do it quickly. Still followed remarks, but kept trying to divide. When sheet with tests was given, tried to regain composure, but still felt excited; which was increased when I failed to retain image of the figures in mind. Kept saying them to myself. When I was writing down results, did not feel at all certain that they were correct; verified two of them by multiplying. Kept referring to sheet to find out what numbers were. Toward the last was conscious of the time element; tried to hurry; kept wondering if most of the students completed them within one minute."

"Twice I felt I was putting down wrong answer, but felt too hurried to go back over problems. Tried to get a system, but in this test could not get any. Saw '*84 by 17*' as *17)84(*. Saw these two as numerals in this order. But with the problems the divisor was over to the right, and I had to bring it over at each operation. Imagery very imperfect; could not hold divisor and dividend in same image; a discontinuous process. Had to bring back by an effort the two different images, both divisor and dividend. Each step involved sense of strain. Distinct motor imagery during the process."

"Saw one specific number at a time. Could not hold entire problem in mind at once."

"Image of figures very vague. I have not as definite an image of figures as I have of straight lines. . . . . The mind's eye moves almost continually in this test, but in a very much broken-up path. I feel a sense of groping that is not present in definite images of geometrical figures. The process is a continual striving

to hold the image of divisor and bring it up to its normal position at left of the dividend. . . . . As process of dividing goes on, a feeling of agitation and confusion comes at dealing with so many things in the same image."

Test 2. Was very tired mentally, as this was the last test. Had been feeling exhilarated, but reaction had set in. When told it was to be division, felt discouraged. Tried to bring images of eight, six and two side by side, but tried several times before I could see them related; then looked at divisor—three; compared it with eight, had no difficulty in seeing six, or in dividing with two as result, but it was difficult to remember the result of subtraction, so repeated the process. Then had difficulty in imaging the figure six, and then recalling result, comparing them and combining them in one image—then forgot divisor, so had to refer to paper to find dividend."

"The whole problem was to picture entire numbers, so that the second or third numbers of dividend could be recalled when needed." . . . "There was no time to get a systematic method started, for in our agitation at the pressure of time we hurry into it without such preparation." . . . . "A very confusing test because the imagery process is so badly broken up. Feel strain always in trying to recall divisor."

"The process is decidedly one of fluctuating attention, a constant fading out and reviving of the image. Both division tests seem more complicated to me than the other tests. The images are indefinite and there are many of them. Test 2 is more difficult than Test 1 because of having three or four figures in the dividend." . . . . "Very difficult to translate written words into images of numerals."

Test 3. "First, an effort to image the letters quickly; second, a little resistance that I saw so unclearly; third, an effort to add mentally without blurring the image of the letters through divided attention. Felt a distinct jolt each time as I moved from one letter to another, not nearly so evident, however, as in division tests. Much easier than division imagery, not so complicated figures to deal with."

"Image of the whole word never appeared. Kept repeating sound of word; then imaged each letter. Much less strain in this test. Required little mental effort, compared to first test, to bring up letters as images." . . . . "While making alphabet, started to count lines, then stopped. *B* and *S* bothered me; during the instructions which followed these two images kept appearing insistently, wanting to be counted; felt that it was a force outside myself presenting these and if I could only stop and count them they would not appear. Forgot how to form the letter *h*, but decided that whether it was *H* or *h* there would be three lines, so sometimes imaged it one way and sometimes another." . . . . "Much less strain—did not feel so hurried—may be partly that I

am getting used to taking the tests." . . . . "Required little mental effort compared with first test to bring letters as images." . . . . "Got on to a system. Learned to begin with first line of letter to left, then at top, then went down." . . . . "Felt that I was solving much more easily, correctly and rapidly; gave a feeling of satisfaction. Was disappointed when time was up." . . . "First imaged whole letter, then that image disappeared, and each line was brought up separately to consciousness. Related to others when whole image would appear; then next line. No image at all of written words. Memory purely auditory-motor. Much less effort than in Test 1, because had to hold but one image in consciousness at once. Test 1, on other hand, required bringing into consciousness two separate images, comparing them and then doing the work, or manipulating them. Both were plane imagery, but entirely unlike."

"First a blackness, then the letters stand out clearly. Only one letter of the word appears at focus of consciousness. Counting of strokes goes almost automatically, although a slight break and a tendency to confusion comes between letters as the image of next letter has to be revived. Counting causes only a slight shift in attention, much less than in division tests."

"This is much easier than division tests; progress much more nearly continuous. There is not so much fluctuating and fading out of the images."

Test 4. "Had to adjust myself mentally to problem. Read it three times. Felt that I could not solve the problem. First conscious that I had to cut the cube, then I was conscious that it was painted—color was blue. Then comprehended that answers depended upon the number and the color. Saw cube distinctly, painted light-blue with lines running as a drawing, not as a solid. Then, when I began to count, I saw it as a drawing, and only realized that it was a solid when I had to count the corner I could not see in the drawing—then it stood out as a solid. In second operation counted laboriously the cubes, top edges—more rapidly those around the sides; then added up the ones in the base—saw these little cubes with but two sides painted. Reasoned out that there would be one cube to each face with one side painted, so did not image cubes after the first one, but added up the faces of the cube instead. By process of elimination reasoned that there would be but one unpainted. Saw it plainly, light tan wood—felt sorry for it." . . . . "Method required in this test; less mental effort; more difficult than third test, owing to solid image and in comparison with plane surfaces."

"Very difficult to get oriented in the problem. Transition from plane figures to the cube was troublesome. After first baffling feeling, the image stood out clearly and the solution came quite easily."

"Changing from plane to plane is difficult and there is considerable difficulty in holding entire cube in mind. In reviving image of the cube each time, a feeling of uncertainty comes as to which

cubes have been counted. The hesitation is not so great as in the division tests. The solid is so definite, it is easier to picture."

"I always moved around the cube the same way—horizontally. I moved from one edge to another vertically first, later horizontally. Harder than the word test. I could not hold so many blocks and lines. In counting, lost track of some."

Test 5. "Saw wedge as a drawing first; only as a solid when I had to count lines not on paper. Saw triangles and squares as a drawing (on one plane). Counted first the lines of the square, beginning at top and going down around to the right; then counted remaining lines bounding top triangle. Then added 2 to result as I imaged separately the triangles going round to the right and down. Image was in blue lines on white paper."

"Saw the box as oblong, about 6 inches by 2 inches. Saw first lid, then box. In counting, counted around top of box first; looked down inside box, but it was too indefinite—too shadowy to see plainly, but have a distinct impression of peering down in. Then counted upright lines, then base lines, added to result; saw line joining the lid; then the lid—then counted three remaining lines and added."

"Saw the triangles and squares as a whole figure; first counted lines of triangle. Then saw right-hand square, counted remaining lines, 8; added them to first, then repeated image in different position and added each time. Felt that I was getting system."

"Counted lines of prisms, then of prism and added. Did not go around object, but reasoned out number of lines, then added."

"Solid figures more difficult than plane figures. See solid first as a whole, then image the separate lines."

"Saw the triangles as a whole. Comparatively few things to image and all seemed well centered—systematized. Feel that they would 'stay put'—whereas the more complicated objects would not. Less confusion."

"Always see the solid first, then break it up into surfaces, then into lines, which I count. As long as I can image on the same plane, there seems almost no break. The transition troubles especially, because the motion is from one plane to another. When one face of the solid is in mind, the others fade out and have to be constantly revived."

B. Imagery Factors Active in the Solution of Problems in Descriptive Geometry.

Nearly every problem of the course involved images of one or more planes, although practically every problem is worked by reference to lines; that is, the line seems to

be the image 'unit.' The planes are always seen bounded by lines, generally 'tied' into other elements of the problems by their 'traces' or intersections with the co-ordinate planes or other inclined planes. There is a sort of definite organization about the imagery content of every problem. Lines and planes are always tied together in a more or less coherent way.

There are no definite counting processes in the descriptive geometry problems. Each problem involves the movement of elements in the visual field. The lines and planes are passed in space, not fixed to a definite position. There is constant motion of the elements, called "revolution," in which lines and planes move about various axes. This movement of elements is always of an organized sort—I mean there is a general effect of smoothness in working and little sense of the break or confusion so frequently spoken of in the tests for mental manipulation.

In résumé, in practically all descriptive-geometry problems: (1) there is a visual fixation of several elements at the same time (points, lines and planes) in various angular positions in space; nearly all problems contain many elements and give practice in fixing several things in the same image and in maintaining and manipulating such elements. (2) The diversity of elements in the image is accompanied by a certain synthesis of organization through many connecting links, which aids in the reviving of needed elements. (3) The factor of systematic movement of the elements of the image, characteristic of so many problems, seems to lessen the strain of concentration on, or of reviving, the image. Movement is thus of a

continuous and unbroken nature, and is nearly always expressed in terms of revolution around some fixed axis.

C. Summary of the Specific Processes Involved in the Solution of Both Test Problems and Descriptive Geometry Problems.

Thus, the training in mental manipulation received in descriptive geometry includes an emphasis on (1) the *focalization* of many geometrical spatial elements in the same image, (2) the *systematic organization* of the many elements of the image as aids to maintaining all in consciousness and to readily recalling those needed, (3) the *continuous movement* of the component elements of the imagery content. The latter two clearly operate to decrease feelings of strain and confusion, and all three factors are effective in developing attitudes of confidence in dealing with complex imagery material of a geometrical nature.

What does a résumé of introspections of students on transfer (in answer to questions on the methods of working the tests) and of the other subjects bring out concerning the imagery processes involved in each test? The essential facts will be presented in condensed form, with some interpretative comment.

Tests 1 and 2. Relatively complex images, containing a large number of separate steps; 90 per cent of the subjects do not use the multiplication tables, but image each specific step in the dividing process. [It should be noted that those using the automatic devices of the tables are those ranking higher in general ability in the descriptive-geometry course.] Test 2 is believed to be harder than Test 1, because of increased number of elements and in-

creased number of visual steps. The process is decidedly discontinuous—movement of the images is very disorganized and not systematic. Images so diverse as to be very difficult of focalization and retention. Considerable evidence of attempts to organize methods of solution. Some subjects developed "system" unconsciously. Auditory and motor imagery quite prevalent. Pressure of time emphasizes the experimental nature of the problem. Recognition of complexity brings on "panicky" feelings of agitation—described as "like the examination feelings of the classroom." Feelings of discomfort and inability were prevalent. Images of specific elements were indefinite.

Test 3. Fewer elements involved in the process. Less strain and less sense of break between the letters than between the steps of Tests 1 and 2. Strain seems to increase with the number of elements. Very few elements can be held in focus at once. Three lines seems to be the limit. Limited range of attention. *Those subjects having the greatest range seem to be the more efficient.* Greater continuity and organization of the path of the mind's eye. Plane imagery seems to offer less difficulty. Definiteness of mechanical letters resulted in much greater facility in focalizing and manipulating images. Images are more concrete than the figures of Tests 1 and 2. Tests 1, 2 and 3 are all of single-plane imagery, but the content and the processes involved are distinctly unlike. Test 3 is regarded on the whole as difficult, because it contains less familiar material for imagery and a large number of elements.

Test 4. Much more complex in analysis. Considerable feeling of discontinuity of movement of mind's eye in changing from one plane to another. The image is lost

more easily at those breaks, and has constantly to be revived. There seems to be a distinct difference in speed and continuity with which imagery on one plane is contrasted with three-dimensional imagery. Imagery of three-dimensional objects (of a geometrical character) is foreign to the customary imagery-content of those persons not working or studying in technical fields. To the technical student there is an easy adjustment to such a geometrical content. Two-thirds of the subjects regard Test 4 as more difficult than 1, 2, or 3, although the outline of the image of the cube is quite concrete and distinct.

Test 5. There are large individual differences in tendencies toward two- and three-dimensional imagery; some subjects, wherever possible, see things as solids; others invariably picture them on one plane (as in pictorial views). Less difficulty is found with more common objects than with less common objects, like the pyramid and the prism. Those tests are easier to focalize and manipulate which have fewer elements. The thing at the focus of consciousness is always the 'working unit,' the line. In all tests except 1 and 2 this unit is definite and easily focalized. Solids and planes are always broken up into lines. Rapidity and accuracy of manipulation are facilitated in a content that is well organized and tied together by clearly marked lines. Difficulty seems to depend on degree of familiarity and on the increase in number of elements in the visual field. A peculiar tendency is evident to image in three-dimensional positions many plane figures which contain diverse combinations of elements, *e. g.,* the triangles and squares of Test 5. Subjects feel that the image can be made more definite in that way.

D. Generalized Factors Present in Several Tests.

In addition to the specific factors mentioned above, there have been developed in the training course certain generalized factors that are closely linked together in operation and that clearly function in many or all of the tests in the case of the training subjects. These may be classified as follows:

(1) The building up of a large number of imagery concepts to which the subject can react with his specific images, *e. g.*, many definite image-concepts of combinations of lines and planes in space, a large and yet particularized background of geometrical concepts—lines, planes, solids, triangles, squares, prisms, etc.

(2) Attitudes of orientation (*a*) in the field of general visual manipulation, (*b*) in the field of specific visual manipulation.

(3) Feelings of familiarity with the materials or content of imagery.

(4) Specific habits of initial adjustment to visual problems. In addition, the actual working of the test problems is complicated by the setting up of general feelings of agitation and strain, the 'examination' feelings that have been mentioned, feelings of excitement in the actual working of the problems, which are accentuated by pressure or hurry due to the time element in the test taking.

E. Analysis of the Process of Mental Manipulation of Spatial Elements and Résumé of the Effective Agencies of Transfer.

Having brought out above the various factors, both specific and general, involved in the process of visual manipulation, let us state briefly the steps involved in the

process itself, endeavoring to find out in just what way the training course has contributed practice in any of these steps. The process involves first, the *particularizing of the concept* of the thing in question; second, a *reaching out for specific 'cues'* to which the individual can respond; third, a resistance to outside influences (a negligible factor if a high degree of attention is secured), and fourth, a 'flashing-up' of the image. The formation of the concept is a function of the range and accuracy of previously acquired knowledge. The rapidity and accuracy of adjustment from concept to specific situation is, in an important measure, a function of the efficiency of the specific visualizing abilities of the subject and of the intensity of attention; it also involves certain attitudes of orientation in the general visualizing field. These attitudes depend largely on familiarity with the imagery content.

The training course has built up a number of geometrical concepts used in the solution of descriptive geometry problems. It has given practice in reacting to specific geometrical 'cues,' and has been effective in establishing certain definite visualizing tendencies. This, in itself, has tended to eliminate the phase of 'groping' or 'feeling' for the adjustment, and has made this adjustment largely habitual and well-nigh instantaneous. The course has developed a thorough feeling of familiarity with the geometrical materials or content found in the geometrical tests, and has developed through constant practice an habitual attitude of orientation in the general field of visualization. We might almost say that there has been formed a 'skill' of visualizing, a quickness and accuracy in response that operates more efficiently in the second test series than in the first.

Thus we conclude that the agencies of transfer in the training of abilities in the mental manipulation of spatial

elements are four in number: (1) Many specific adjustments or reactions to familiar cues of visualization have tied together training and test series and have undoubtedly acted as agencies through which more efficient response is made to new situations. (2) An important phase of the successful response to a new situation is the building up of attitudes of orientation in the general visual field. (3) Practice in extending the range of attention has given increased facility in holding and manipulating a large number of visual elements at the same time. (4) The study of the effect of training upon various grades of scholastic ability pointed to the probable effectiveness of conceptualizing abilities in developing methods of analysis or attack. The introspective material confirms the view that these conceptualizing abilities play an important part in determining the method of transfer (witness the many references to the attempts "to get a system;" to develop a method, etc.).

That the foregoing agencies are active in both test and training series is evident. Just in what degree and in what measure they are related we cannot definitely say; but that they do account for the residual gains on the part of the trained subjects there is no longer any doubt in the mind of the writer.

F. Further Interpretive Comment.

(a) The Influence of Certain Experimental Conditions in the Study.

Even though this investigation was carried on under as nearly normal classroom conditions as it was possible to obtain with unusual precautions, there is evidence that the 'time' element furnished a cause for distraction and

strain peculiar to the laboratory type of educational psychology experimentation. We have still far to go in our search for methods of classroom testing of specific abilities which will eliminate attitudes or states of mind so unusual or abnormal as these feelings of agitation and excitement. Thoroughly rated tests, to be taken naturally by students, must be a part of the regular organization of the course, like recitations, quizzes, and laboratory work, and they must appear as a recognized phase of the regular work. It was clear that this could not be accomplished in the conduct of this investigation, and the difficulties pertaining to all such classroom testing of ability seem at the present time almost insurmountable. We believe, however, that the unusual or 'experimental' atmosphere has been reduced to a minimum in the giving of these tests. It seems clear that such abnormal conditions as may have been present operated to hinder transfer rather than aid it, so that the statistical results and conclusions from this study cannot be affected by this factor.

(b) Drill in Extending the Range of Attention and Manipulation.

In the problems of the descriptive geometry course considerable training was given in focalizing and manipulating images which involve the holding in mind simultaneously and concurrently many elements. Tests 3 and 6 are so organized that one can focalize one element of the object at a time, count it and drop it from the visual field. In Tests 1 and 2, however, one is compelled to focalize several elements simultaneously or to be constantly reviving those which have faded out. The practice in extending the range of attention so essential to this skill (*i. e.*, increasing the number of elements which can be retained and manipulated at once), and of dealing with

many elements in the image at one time, as obtained in the training course, undoubtedly was effective in the second test series in Tests 1 and 2 as well as in those tests whose content more closely resembles that of the descriptive geometry course.

(c) The Fallacy of the "Geometrical Imagination."

It seems perfectly clear from a study of all of the statistical and introspective data that there can be no such mysterious "ability" as geometrical or non-geometrical "imagination." There are probably a large number of specifically distinct imaginal activities which are predominantly concerned with geometrical elements, and which operate more efficiently in a geometrical 'setting.' It is also undoubted that there is a large number of imaginal activities in which no geometrical elements appear. Efficient response to one non-geometrical situation will be highly correlated with efficient response to other so-called non-geometrical situations, but will not imply a like degree of efficiency in the others. Similarly with geometrical manipulating functions: abilities in these are correlated to a considerable extent, although hardly enough to be called a high relationship ($r = .30$ to $.45$), and will vary largely with the specific content of the situation. In general, our results point to the tremendous complexity of the mental activities involved in the study of school subjects, even in fields so seemingly simple as visual manipulation. It would seem that we should study the efficiency of the learning process in various school subjects by analyzing each complex into its special outcomes. For example, "abilities in plane geometry" should be analyzed into its constituent special outcomes as memory of definitions, power of applying definitions, memory of rules, power of applying principles, accuracy in construction, etc.

CHAPTER XI.

SUMMARY OF CONCLUSIONS.

A. Does Training Transfer?

The study of descriptive geometry (under ordinary class room conditions throughout a semester of 15 weeks) in which such natural and not undue consideration is given to practice in geometrical visualization as is necessary for the solution of descriptive geometry problems operates:

(1) Substantially to increase the students' ability in solving problems requiring the mental manipulation of a geometrical nature, the content of which are distinctly different from the visual content of descriptive geometry itself.

(2) Substantially to increase the students' ability in solving problems requiring the mental manipulation of spatial elements of a slightly geometrical character, *i. e.*, problems utilizing the fundamental elements of geometry (the point, line and plane), but apart from a geometrical setting, and in such form as to offer no geometrical aids in solution.

(3) Substantially to increase the students' ability in solving problems requiring the mental manipulation of spatial elements of a completely non-geometrical nature, *i. e.*, problems in which the straight line and plane do not appear in any way whatsoever.

(4) The training effect of such study in descriptive geometry operates more efficiently in those problems whose visual content more closely resembles that of the training course itself, *i. e.*, in those problems whose imagery content is composed of combinations of points, lines and planes, and

in which the continuity of the manipulating movements approaches the continuity of those in the training course.

B. Does Training Transfer in Like Degree with Various Grades of Scholastic Ability?

If students are grouped on a basis of scholastic ability in 'disciplinary' or 'training' courses, such as the various college courses in mathematics, the semester's training received in descriptive geometry operates more effectively with the high-grade student than with the low-grade student. If subjects are grouped on a basis of scholastic ability in English and modern languages, the training is equally effective with all grades of scholastic ability. There are thus decided implications that the method of transfer is determined largely in terms of 'conceptualizing' abilities.

C. What is the Relative Effect of Training on Adults of Varying Ages?

Within the limits represented by this study (17 to 26 years) the age of the subject does not seem to be a factor in determining the effect of training on his ability in the mental manipulation of spatial elements.

D. What Are the Agencies of Transfer?

It is concluded from this study that the agencies of transfer are the following:

1. Many specific adjustments or reactions to familiar cues of visualization have tied together training and test series, and have undoubtedly acted as bonds or agencies through which more efficient response is made to a given situation.

2. An important phase of the successful response to a new situation is the building up of attitudes of orientation in the general visual field.

3. Practice in extending the range of attention has given increased facility in holding and manipulating a large number of visual elements at the same time.

4. The study of the effect of training on various grades of scholastic ability pointed to the probable effectiveness of conceptualizing abilities in developing methods of analysis and attack. The introspective material confirms the view that these conceptualizing abilities play an important part in determining the method of transfer.

E. The Possibility of "Disciplinary Outcomes" in School Studies.

The possibility of one 'disciplinary outcome' in a specific school subject, *i. e.,* ability in the mental manipulation of spatial elements, has been established in this investigation. The writer believes that formal school subjects find a large part of their disciplinary value in the developing of this ability to analyze the problem and to organize a method of procedure; to build up ideals, or to organize a method of attack. But it is undoubted that they also make habitual, or automatic, many specific constituents of the complex abilities that function in many complex situations. The successful habitualizing of these specific reactions is accentuated by the building up of a background of fundamental attitudes of orientation, or familiarity with the content of the situations to be met. It may be increased by the accompaniment of practice in extending the range of attention.

SELECTED BIBLIOGRAPHY.

(Arranged chronologically according to grouping found on Plates I to III.)

1. *Studies Dealing with Memory Abilities.*

1890. JAMES, W. *Principles of Psychology.* (P. 67.)

1904. EBERT, *E.*, and MEUMANN, E. *Ueber einige Grundfragen der Psychologie der Uebungsphänömene im Bereiche des Gedächtnesses.* Archiv fur die gesamte Psycholigie, vol. 4, pp. 1-232.

1908. FRACKER, G. C. *On the Transference of Training in Memory.* Psychological Review Monograph Supplement, No. 38, pp. 56-102.

1908. WINCH, W. H. *The Transference of Improvement in Memory in School Children. I.* British Journal of Psychology, vol. 2, pp. 284-293.

1909. DEARBORN, W. F. *The General Effects of Special Practice in Memory.* Psychological Bulletin, vol. 6, pp. 44.

1910. WINCH, W. H. *The Transfer of Improvement in Memory in School Children. II.* British Journal of Psychology, vol. 3, pp. 386-405.

1910. DEARBORN, W. F. *Experiments in Learning.* Journal of Educational Psychology, vol. 1, pp. 373-388.

1911. SLEIGHT, W. G. *Memory and Formal Training.* British Journal of Psychology, vol. 4, pp. 386-457.

1912. PETERSON, H. A. *Note on a Retrial of Professor James's Experiment on Memory Training.* Psychological Review, vol. 19, pp. 491-492.

2. *Studies Dealing with Sensory, Perceptual and Associative-Motor Functions.*

1896. JASTROW, J. *Psychological Notes Upon Sleight-of-Hand Experts.* Science, N. S., vol. 3, pp. 685-689.

1897. GILBERT, J. A., and FRACKER, G. C. *The Effects of Practice in Reaction and Discrimination for Sound Upon the Time of Reaction and Discrimination of Other Forms of Stimuli.* University of Iowa Studies in Psychology, vol. 1, pp. 62-76.

1901. THORNDIKE, E. L., and WOODWORTH, R. S. *The Influence of Improvement in One Mental Function Upon the Efficiency of Other Functions.* Psychological Review, vol. 8, pp. 247-261; 384-395; 553-564.

1902. JUDD, C. H. *Practice and Its Effects on the Perception of Illusions.* Psychological Review, vol. 9, pp. 27-39.

1902. BAIR, J. H. *The Practice Curve.* Psychological Review Monograph Supplement, No. 19.

1905. LEUBA, J. H., and HYDE, W. *An Experiment in Learning to Make Hand Movements.* Psychological Review, vol. 12, pp. 351-369.

1907. COOVER, J. E., and ANGELL, F. *General Practice Effect of Special Exercise.* American Journal Psychology, vol. 18, pp. 327-340.

1907. BENNETT, C. J. C. *Formal Discipline.* Teachers' College Contributions to Education.

1908. SCHOLKOW and JUDD, C. H. *The Relation of Special Training to General Intelligence.* Educational Review, vol. 36, pp. 28-42.

1909. KLINE, L. W. *Some Experimental Evidence on the Doctrine of Formal Discipline.* Bulletin of the

State Normal School, Duluth, Minn., February, 1909. More completely described in Journal Educational Psychology, May, 1914.

1910. RUGER, H. A. *The Psychology of Efficiency.* Archives of Psychology, No. 15.

1910. WALLIN, J. E. W. *Two Neglected Instances of the Transfer of Training.* Journal of Educational Psychology, vol. 1, pp. 168-171.

1910. WHIPPLE, G. M. *The Effect of Practice Upon the Range of Visual Attention and of Visual Apprehension.* Journal of Educational Psychology, vol. 1, pp. 249-262.

1911. FOSTER, W. S. *The Effect of Practice Upon Visualizing and Upon the Reproduction of Visual Impressions.* Journal of Educational Psychology, vol. 2, pp. 11-21.

1914. DALLENBACH, K. M. *The Effect of Practice Upon Visual Apprehension.* Journal of Educational Psychology, vol. 5, pp. 321 and 387.

3. *Studies Dealing with School Activities.*

1905. BAGLEY, W. C., and SQUIRE, CARRIE R. *Experiment on Transfer of Ideals of Neatness Reported in* BAGLEY: Educational Values, pp. 188-189.

1907. BENNETT, C. J. C. *Formal Discipline.* Columbia University Contribution to Education.

1908. RUEDIGER, W. C. *The Indirect Improvement of Mental Functions Through Ideals.* Educational Review, vol. 36, pp. 364-371.

1911. WALLIN, J. E. W. *Spelling Efficiency in Its Relation to Age, Sex and Grade.* Educational Psychology Monographs.

1913. BRIGGS, T. H. *Formal English Grammar as a Dis-*

cipline. Teachers' College Record, September, 1913.

APPENDIX.

Further Detailed and Summary Tables.

Tables A-1, A-2, A-3. Efficiency obtained by each subject in Tests 1 to 6.

Tables B-1 to B-20. Shows number of subjects making certain number of attempts and obtaining a certain number of these correct.

Table C. For Tests 1 to 5. Summary of the median number of operations or problems worked and the median gains made by the training and control groups.

Table D. For Test 6. Number of words obtained in both tests series and by both groups; number of words gained and per cent gained.

Table E. For Test 6. Aggregate and average number of words obtained for each of the five mental processes.

Tables F-1, F-2, F-3. For Tests 1 to 6. Per cent gain made by each subject.

Table G. For Tests 1 to 6. Number and per cent of subjects gaining.

Table H. For Tests 1 to 5. Summary of aggregate and average gains.

Table I. Number of subjects gaining in various proportions of tests.

Table J. For Tests 1 to 5. Progress of the training subsections.

Tables K-1 to K-34. Pearson coefficients of efficiency in Tests 1 to 5.

Tables L-1 to L-4. Coefficients of efficiency in tests with ability in descriptive geometry.

Tables M-1 to M-8. Correlation of scholastic ability in various college studies.

TABLE A-1.

VARIOUS DATA CONCERNING SUBJECTS OF INVESTIGATION—(1) NUMBER; (2) TRAINING SUB-SECTION; (3) AGE; (4) GRADES IN (a) DESCRIPTIVE GEOMETRY; (b) MATHEMATICS; (c) SHOP-PRACTICE; (d) ENGLISH; (e) MODERN LANGUAGES; (f) MECHANICAL DRAFTING; (5) EFFICIENCY OBTAINED IN TESTS 1 TO 5, INCLUSIVE (a) FEBRUARY TESTS SERIES; (b) JUNE TESTS SERIES.

(This table is inserted as a sample of the method of tabulation followed in Tables A-1, A-2, A-3. The complete original tabulations may be inspected by those interested by communicating with the writer.)

			Grades No. 4						(5) Efficiency in Test Series													
									(a) February Series							(b) June Series						
1	2	3	a	b	c	d	e	f	1	2	3	4	5	6	6a	1	2	3	4	5	6	6a
25	1	21–10	72	70	75	75	82	82	50	20	58	25	13	12	9	47	45	83	25	50	25	16
26	..	20–5	70	81	88	79	..	87	33	40	25	25	25	..	..	50	50	67	25	25	..	..
30	..	18–4	70	72	85	..	76	..	..	..	58	..	..	..	..	..	48	75	..	50	..	..
31	1	19–2	94	92	92	81	..	89	60	45	67	50	50	26	17	83	45	50	75	50	24	15
32	2	19–11	97	96	87	93	..	94	37	30	58	0	63	22	14	50	55	67	75	38	30	25
33	7	19–9	88	70	76	..	..	88	70	50	67	50	63	26	20	70	60	58	75	88	31	21
34	3	19–3	61	63	84	20	..	87	37	25	58	0	50	16	11	47	40	75	25	50	17	13
35	..	....	62	..	..	..	..	..	28	50	50	38	..	..	..	..	..	..	..	..	..	..
36	3	26–6	89	80	89	..	55	88	47	50	67	25	50	9	7	60	43	58	75	50	12	10
37	2	19–5	94	96	93	..	93	93	50	40	42	50	13	20	15	60	40	67	50	38	28	20
38	2	20–9	94	91	88	..	75	91	67	50	67	100	50	23	18	67	53	50	75	75	32	24
39	..	19–9	44	50	74	..	84	73	73	55	58	0	50	25	19	27	48	50	25	38	25	19
40	2	21–7	87	81	85	..	70	87	23	50	42	25	50	24	23	77	50	42	50	75	24	20
41	3	22–3	78	85	83	..	80	..	13	25	42	0	25	11	10	40	30	25	0	13	20	16
42	2	17–9	73	76	83	..	84	85	17	28	58	25	38	16	15	30	33	58	50	63	19	11
43	7	19–8	88	88	85	57	..	84	20	50	42	0	25	3	2	57	48	58	50	50	15	13
44	..	....	..	..	..	..	..	..														

TABLE B.

Efficiency of Training Group – Test 3

Table B-9 February.

	0	1	2	3	4	5	6	7	8	9	10	11	12	
12	Median number operations													
11	attempted 6.479											1		1
10	right: 4.667										1	3	1	5
9										3	3		1	7
8									23	9	2			34
7								20	14	3	1			38
6							17	18	9	2	1			47
5						4	10	11	6	2				33
4					4	13	12	8	9	1				47
3				1	6	18	7	3	6	1				42
2				5	6	3	2		2					18
1			1	1	4	3		1						10
0	1				1	1			1					4
	1		1	7	21	42	48	61	70	21	8	4	2	286

Table B-10 June.

	0	1	2	3	4	5	6	7	8	9	10	11	12	
12	Median number problems												1	1
11	attempted 7.527. Gain											5	1	6
10	= 1.048										9	4	2	15
9	Median number problems right:									7	8	3	3	21
8	5.659. Gain = 0.992								9	9	6	2	1	27
7								13	25	9	5	3	1	56
6							2	16	13	8	3		2	44
5						1	9	11	11	5	1	3		41
4					3	7	4	9	12	2	1			38
3					4	4	4	2	3	1	1			19
2					1	4	1							6
1					1	5		1	1					8
0														
					9	21	20	52	74	41	34	20	11	282

Efficiency of Control Group – Test 3

Table B-11 February

	0	1	2	3	4	5	6	7	8	9	10	11	12	
12	Median number problems													
11	attempted: 6.211											1	1	2
10	right: 4.500													
9										1				1
8									2	1				3
7								5	4					9
6							6	11	1	2				20
5						2	3	5	4	1				15
4					2	3	1		1				1	8
3				4	2	5	1	3						15
2				1	3		1	2		1				8
1			1			1				1				3
0					1									1
			1	5	8	11	12	26	12	7		1	2	85

Table B-12 June

	0	1	2	3	4	5	6	7	8	9	10	11	12	
12	Median number problems													
11	attempted: 6.853. Gain =													
10	= 0.642.										1			1
9	Median number problems right: 4.583									3	2			5
8	Gain = 0.083								2	2	1	1	1	7
7								4	3	1				8
6							4	3	6	3				16
5						1	3	4	4					12
4					1	2	3	4		4		1		15
3						3	3	1	1	2				10
2					2	1	2	1	1					7
1				1	2									3
0														
				1	5	7	15	17	17	15	4	2	1	84

These tables are typical of Tables B-1 to B-20, each of which contains data similar to those above. All original tabulations have been filed by the writer and may be inspected by those interested.

TABLE C.

MEDIAN NUMBER OF PROBLEMS OR OPERATIONS WORKED BY ALL SUBJECTS IN TESTS 1 TO 5, INCLUSIVE.

Figures Give Median Number Attempted, Number Right, and Per Cent Gained.

Training Group.

	Test 1				Test 2			
	Attempts		Rights		Attempts		Rights	
	No.	Gain	No.	Gain	No.	Gain	No.	Gain
February...	24.08	....	20.00	....	23.72	....	20.00	....
June........	25.39	1.21	22.81	2.81	24.93	1.21	23.00	3.00
Per Ct. Gain	.....	5.0	.....	14.5	.....	5.5	.....	15.0

	Test 3				Test 4			
	Attempts		Rights		Attempts		Rights	
	No.	Gain	No.	Gain	No.	Gain	No.	Gain
February...	6.48	....	4.67	....	2.91	....	0.59	....
June........	7.53	1.05	5.66	6.90	3.35	0.44	1.50	0.91
Per Ct. Gain	....	16.1	....	21.2	....	15.0	....	157.0

	Test 5			
	Attempts		Rights	
	No.	Gain	No.	Gain
February....	3.95	....	2.32	....
June.........	4.93	0.98	3.33	1.01
Per Ct. Gain.	....	24.6	....	43.5

Control Group.

	Test 1				Test 2			
	Attempts		Rights		Attempts		Rights	
	No.	Gain	No.	Gain	No.	Gain	No.	Gain
February...	20.06	....	17.38	....	20.92	....	19.3	....
June........	20.79	0.73	20.06	2.68	20.96	0.04	19.15	0.15
Per Ct. Gain	.....	3.6	.....	15.6	.....	0.05	.....	—0.78

	Test 3				Test 4			
	Attempts		Rights		Attempts		Rights	
	No.	Gain	No.	Gain	No.	Gain	No.	Gain
February...	6.21	....	4.50	....	2.55	....	0.28	....
June........	6.85	0.64	4.58	0.083	3.15	0.60	0.68	0.40
Per Ct. Gain	....	9.7	....	1.8	....	23.2	....	143.0

	Test 5			
	Attempts		Rights	
	No.	Gain	No.	Gain
February....	3.44	....	1.92	....
June.........	3.77	0.33	1.81	—0.10
Per Ct. Gain.	....	9.6	....	—5.0

(Training group data are (for Tests 1 and 2) for 1914 only, those for 1913 not being strictly comparable with those of the control group. Not obtained in precisely the same way.)

TABLE D.

CONCERNING THE NUMBER OF WORDS OBTAINED IN THE SOLUTION OF TEST 6.

TRAINING GROUP.

1913 Training Group.

Section	No. of Subjects	No. of Words Obtained February	June
A-2	16	314	394
A-3	20	378	468
B-1	22	430	534
B-2	16	273	365
B-3	17	335	387
C-2	22	410	525
C-3	18	327	392
D-1	15	254	320
D-2	16	300	419
D-3	16	348	407
	178	3,369	4,211
			3,369
Number of words gained			842
Per cent of words gained			24.9

1914 Training Group.

Section	No. of Subjects	No. of Words Obtained February	June
A-1	16	233	316
A-4	18	398	440
B-1	15	306	368
B-4	9	202	236
C-1	18	329	419
D-1	15	299	402
D-2	13	248	329
	104	2,015	2,510
			2,015
Number of words gained			495
Per cent of words gained			24.6

SUMMARY: ENTIRE TRAINING GROUP.

Total Number of Words Obtained.

	February	June
	5,384	6,721
		5,384
Number words gained		1,337
Per cent words gained		24.87

Average Number Words Obtained per Training Subject.

February	June
191.11	23.9

Number and Per Cent of Subjects Gaining in Test 6.

	No. Gaining	No. Not Gaining	Per Cent Gaining
1913 Group	140	38	78.6
1914 Group	85	19	81.7
	225	57	80.0

CONTROL GROUP.

Group No.	No. of Subjects	No. of Words Feb.	No. of Words June	No. of Words Gained	Per Cent Words Gained
1	15	256	310		
2	34	756	907		
3	32	716	911		
	81	1,728	1,928	200	11.5
			1,728		
			200		

Average Number of Words Obtained per Control Subject.

February	June
21.3	23.8

Number and Per Cent of Control Subjects Gaining.

No. Gaining	No. Not Gaining	Per Cent Gaining
67	16	80.7

TABLE E.

DATA CONCERNING THE METHODS BY WHICH THE SUBJECTS OBTAINED THE WORDS IN TEST 6.

Number of Words Obtained by Each of the Five Psychological Processes.

1913 Training Group.

February Section					June Section				
A	O	S	V	I	A	O	S	V	I
772	297	77	2,372	61	880	398	139	2,950	67
880	398	139	772	67				880	
1,652	695	216	3,144	128				3,830	
			3,830						

Total of 6974 words obtained by using A and V.

1914 Training Group.

February Section					June Section				
A	O	S	V	I	A	O	S	V	I
496	135	227	1,321	38	693	201	255	1,583	41
693	201	124	1,583	41					
1,189	336	351	2,904	79					
			1,189						

1914. Total of 4,093 words formed by using A and V.
1913. Total of 6,974 words formed by using A and V.

Grand Total of 11,067 words formed by using A and V.
1936 words obtained by entire training group by using O, S and I.

Number of Words Obtained by Using V Alone.

Year	February	June	Gain	Per Cent Gain
1913......	2,372	2,950		
1914......	1,321	1,583		
	3,693	4,533	840	22.8 gained in using V alone.
		3,693		

Control Group, 1914.

February Section					June Section				
A	O	S	V	I	A	O	S	V	I
451	177	131	1,205	23	590	192	152	1,388	41
590	192	152	1,388	41				1,205	
1,041	369	283	2,593	64				183 words gained, or 15.4% gained using V alone.	
	283		1,041						
	64								
			3,634 words obtained using O, S and I.						
	716 words obtained using O, S and I.								

A = auditory imagery.
O = organization of letters.
S = systematic method.
V = visual imagery.
I = ideational types.

For detailed explanation of method of scoring this test see Chapter 8, on the scoring of Test 6.

TABLE F-1.

PER CENT GAIN OF EACH SUBJECT IN "ATTEMPTS" AND "RIGHTS," TESTS 1 TO 5, INCLUSIVE.

(x signifies a constant record, no gain and no loss.)

RIGHTS.

	Per Cent Gain						Per Cent Loss				
No.	1	2	3	4	5	% G.	1	2	3	4	5
25........	..	..	43	..	300	2/4	6	..	..	x	..
26........	50	25	166	..	..	3/5	..	..	..	x	x
30........	..	..	29	..	..	1/1	..	..	..	..	..
31........	40	..	..	50	..	2/5	..	x	25	..	x
32........	40	83	14	300	..	4/5	..	..	..	..	40
33........	..	20	..	50	40	3/5	x	..	12	..	..
34........	30	60	29	100	..	4/5	..	..	..	..	x
36........	30	..	..	200	..	2/5	..	15	12	..	x
37........	20	..	60	..	200	3/5	..	x	..	x	..
38........	..	5	..	..	50	2/5	x	..	25	25	..

ATTEMPTS.

	Per Cent Gain						Per Cent Loss				
No.	1	2	3	4	5	% G.	1	2	3	4	5
25........	..	150	43	25	25	4/5	12	..	..	..	..
26........	25	..	60	100	..	3/5	..	x	..	40	..
30........	..	..	12	..	..	1/1	..	..	..	..	..
31........	20	..	..	..	20	2/5	..	x	12	x	..
32........	..	7	29	33	..	3/5	x	..	..	..	29
33........	33	20	25	8	..	4/5	..	..	..	x	..
34........	33	..	12	100	20	4/5	..	x	..	..	..
36........	20	5	12	..	..	3/5	..	..	12	..	x
37........	40	5	..	20	25	4/5	..	..	x	..	..
38........	..	20	..	45	75	3/5	x	..	12	..	..

(This table is inserted here as a sample of the original tabulation of "Per Cent Gains" of the various subjects. Tables may be inspected by those interested by communicating with the writer.)

TABLE G.

NUMBER AND PER CENT OF SUBJECTS GAINING IN TESTS 1 TO 5.

"ATTEMPTS."

	Number				Per Cent			
	Training Group			Control Group	Training Group		Control Group	Residual Gain
Test	1913	1914	Total	1914	1913	Total	Total	
1. gain..	136	78	214	40	65.5	69.2	47.6	21.6
even..	32	8	40	19	15.7	....	....	....
loss..	39	17	56	25	18.8	30.8	52.4	....
2. gain..	112	79	191	39	58.2	64.2	45.3	18.9
even..	28	3	31	17	14.5	....	....	....
loss..	53	23	76	30	27.3	35.8	54.7	....
3. gain..	134	51	185	54	65.0	64.6	63.5	1.1
even..	48	13	61	16	23.0	....	....	....
loss..	25	16	41	15	12.0	35.4	36.5	....
4. gain..	122	67	189	36	63.4	63.7	43.7	20.0
even..	57	30	87	30	29.5	....	....	....
loss..	14	7	21	16	7.1	36.3	56.3	....
5. gain..	125	71	196	39	64.6	66.0	45.8	20.2
even..	42	20	62	30	21.6	....	....	....
loss..	27	13	40	16	13.8	34.0	54.2	....

"RIGHTS."

Test	1913	1914	Total	1914	1913	1914	Total	Control Total	Residual Gain
1. gain..	151	78	229	40	73.1	76.0	74.0	47.6	26.4
even..	22	8	30	19	10.5	....	....	....	....
loss..	34	17	51	25	16.4	....	....	....	....
2. gain..	121	79	200	34	63.0	75.0	67.8	39.5	28.3
even..	16	3	19	10	8.0	....	....	....	....
loss..	56	23	79	42	29.0	....	....	....	....
3. gain..	114	51	165	38	55.2	63.7	57.6	44.2	13.4
even..	39	13	52	19	18.7	....	....	....	....
loss..	54	16	70	29	26.1	....	....	....	....
4. gain..	97	67	164	31	50.3	64.0	55.3	37.8	17.5
even..	66	30	96	34	34.2	....	....	....	....
loss..	30	7	37	17	15.5	....	....	....	....
5. gain..	115	71	186	30	59.5	68.2	62.5	35.3	27.2
even..	38	20	58	28	19.5	....	....	....	....
loss..	41	13	54	27	21.0	....	....	....	....
6. gain..	140	85	225	67	78.6	84.7	80.0	80.7	—0.7
even..	38	19	57	16	21.4	....	....	....	....

TABLE H.

AGGREGATE AND AVERAGE GAINS IN TRAINING AND CONTROL GROUPS, TESTS 1 TO 5, INCLUSIVE.

TEST 1.

	Attempts				Rights			
		No.	Per Cent			No.	Per Cent	
Group	Aggregate Gain	of Subjects	Average Gain	Residual Gain	Aggregate Gain	of Subjects	Average Gain	Residual Gain
1913 Tr.......	774	136	5.69	....	836	151	5.54	....
1914 Tr.......	372	60	6.21	....	427	78	5.49	....
Total.......	1,146	196	5.85	....	1,263	229	5.52	....
Control.......	222	40	5.56	5.72	238	40	5.96	—8.7

TEST 2.

	Attempts				Rights			
1913 Tr.......	612	112	45.46	....	559	121	4.62	....
1914 Tr.......	361	71	5.08	....	385	79	4.9	....
Total.......	973	183	5.32	....	944	200	4.71	....
Control.......	169	39	4.34	22.6	126	34	3.71	21.2

TEST 3.

	Attempts				Rights			
1913 Tr.......	270	134	2.04	....	272	114	2.393	....
1914 Tr.......	135	161	2.22	....	135	51	2.65	....
Total.......	405	195	2.077	....	407	165	2.47	....
Control.......	92	54	1.705	21.8	77	38	2.03	21.2

TEST 4.

	Attempts				Rights			
1913 Tr.......	164	122	1.35	....	146	97	1.52	....
1914 Tr.......	66	37	1.78	....	123	67	1.84	....
Total.......	230	159	1.445	....	269	164	1.64	....
Control.......	57	36	1.58	—10.0	47	31	1.52	7.9

TEST 5.

	Attempts				Rights			
1913 Tr.......	240	125	1.92	....	250	115	2.17	....
1914 Tr.......	165	78	2.12	....	157	71	2.21	....
Total.......	405	203	2.00	....	407	186	2.19	....
Control.......	63	39	1.615	24.2	45	30	1.50	46.0

TABLE I.—NUMBER OF SUBJECTS WHO GAINED IN VARIOUS PROPORTIONS OF TESTS TAKEN.

Group	All 5	All 4	All 3	All 2	All 1	4/5	3/4	2/3	3/5	Total 3/5 and above	2/4	1/2	2/5	1/3	1/4	1/5	Total 1/5 to 3/5	0/1	0/2	0/3	0/4	0/5	Total
RIGHTS.																							
Training Groups.																							
1913........	12	2	1	12	3	42	7	..	71	150	9	1	34	1	4	13	62	3	1	..	..	1	5
1914........	12	5	3	..	1	26	12	1	27	87	9	1	5	..	4	1	20	1	..	..	..	1	2
Total.....	24	7	4	12	4	68	19	1	98	237 72.7%	18	2	39	1	8	14	82 25.1%	4	1	..	..	2	7 336 2.2%
Average number tests in which gains were made = ⅔ or $\frac{38.6}{60}$ = 64.5%.																							
Control Groups.																							
No. 1........	..	..	..	..	1	..	..	..	3	4	..	..	5	..	..	4	9	..	..	..	..	2	2
No. 2........	..	..	..	..	2	..	1	..	7	10	3	..	13	..	..	8	24	..	..	..	1	1	2
No. 3........	..	..	..	..	3	..	..	..	10	13	2	..	7	2	1	8	20	..	..	..	..	3	3
Total.....	..	..	..	..	6	..	1	..	20	27 31.0%	5	..	25	2	1	20	53 61.0%	..	..	..	1	6	7 87 8.0%
Average number tests in which gains were made = ⅖ or $\frac{24}{60}$ = 40.0%.																							
ATTEMPTS.																							
Training Groups.																							
1913........	17	4	1	6	4	53	12	..	49	147	5	5	42	1	4	10	67	2	2	..	..	..	4
1914........	14	3	1	..	1	28	9	2	26	84	6	1	11	1	8	4	31	1	..	..	2	..	3
Total.....	31	7	2	6	5	81	21	2	75	231 67.8%	11	6	53	2	12	14	98 30.1%	3	2	..	2	..	7 336 2.1%
Average number tests in which gains were made = ⅔ or $\frac{37.7}{60}$ = 63.0%.																							
Control Groups.																							
No. 1........	..	..	..	..	..	2	..	..	3	6	..	..	6	..	..	3	9	..	..	..	..	..	..
No. 2........	..	..	..	..	..	7	1	..	8	18	1	1	8	..	3	4	17	..	..	..	2	..	2
No. 3........	..	..	..	..	..	5	3	..	3	13	1	..	8	1	1	10	21	..	..	..	1	..	1
Total.....	..	..	..	..	..	14	4	..	14	37 42.5%	2	1	22	1	4	17	47 54.0%	..	..	..	3	..	3 87 3.5%

Average number tests in which gains were made = ½ or $\frac{29.3}{60}$ = 49.0%.

TABLE J.—COMPARISON OF PROGRESS OF SUBJECTS IN TRAINING SUB-SECTIONS WITH THE SUBJECTS OF SUBJECTS IN ENTIRE TRAINING GROUP. TESTS 1 TO 5, INCLUSIVE.

Per Cent of Subjects Gaining; Average Per Cent Gain of the Subjects; Deviations.

	Per Cent Gaining							Average Per Cent Gain						
	Trg.	Training Sub-Sections						Trg.	Training Sub-Sections					
	Grp.	1	2	3	4 and 5	6	7	Grp.	1	2	3	4 and 5	6	7
Test 1	74	82	69	88	65	83	80	50.7	46	55	64	34	44	56
Deviation of Sub-Training Sections from entire Training Group		8	—5	14	—9	9	6		—4.4	5.7	13.7	—16.8	—6.7	5.3
Test 2	68	62	61	100	85	58	83	36.2	36	43	52	24	38	32
Deviations of S. T. Sections		—6	7	32	17	—10	15		—0.4	6.5	16.0	—12.2	1.7	—3.8
Test 3	58	58	61	58	32	66	56	79.8	106	70	32	22	101	50
Deviations of S. T. Sections		0.0	3	0	—26	8	—2		26.2	—8.4	—47.8	55.4	20.2	—29.8
Test 4	55	64	55	67	50	55	59	146.8	151	149	150	118	129	171
Deviations of S. T. Sections		9	0	12	—5	0	4		4.2	2.2	3.2	—27.2	—17.8	24.2
Test 5	63	73	63	43	65	61	60	116.2	108	126	78	94	148	98
Deviations of S. T. Sections		10	0	—20	2	—2	—3		—8.2	9.8	—38.2	—22.7	31.8	—18.2

Number of Subjects in Various Sub-Sections Taking Certain Tests:

	1	2	3	4 and 5	6	7
Test 1	55	116	8	20	60	34
Test 2	50	119	6	20	57	35
Test 3	54	101	7	17	55	34
Test 4	50	112	6	20	57	36
Test 5	52	110	7	20	56	37

TABLE K.

Correlation of Efficiency of Trained Subjects in Tests 1 and 2 (June) Test 2

Test 1

	0-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70	71-75	76-80	81-85	86-90	91-95	96-100				
100-96									1	1	1	3	3	2	3	4	3	3	3		24	7	168	1176
95-91										1			2		1						5	6	30	180
90-86								1		2	5	1	3	4				1			17	5	85	425
85-81									1	1	4	1									7	4	28	112
80-76							1	2	5	4	8	5	2	1	2						30	3	90	270
75-71								1		1	2	2	4								10	2	20	40
70-66						1		5	9	5	10	11	6	1							48	1	48	48
65-61						1	1	2	1	2		1									8	0	469	
60-56					1	1	4	7	10	9	4		1	2							39	-1	-39	39
55-51								2	2	1	2	1									8	-2	-16	32
50-46					1		3	10	10	5	6	2	1								40	-3	-120	360
45-41				1	2		4	2	3	1			1								14	-4	-56	224
40-36		9		1	1	4		3	2	3	1	2									18	-5	-90	450
35-31					2	1	1	3		3		1									11	-6	-66	396
30-26					1				1	3											5	-7	-35	245
25-21			9																		1	-8	-8	64
20-16	9			1		1															3	-9	-27	243
15-11				1			1														2	-10	-20	200
10-6																							-477	4504
5-0																							c=.003	
	1	1	1	4	8	9	17	38	45	42	38	30	28	10	6	5	2	3	2		290			
	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9					
	-9	-8	-7	-24	-40	-36	-51	-76	-45 (-296)		38	60	84	40	30	30	14	24	18 (338)	c=.145				
	81	64	49	144	200	144	153	152	45		38	120	252	160	150	180	98	192	162 (2384)	σ=2.88	σ^2=8.22 -.02 = 8.20			

Line of Regression

$\sigma_y^2 = 15.53$

$\sigma_y = 3.94$

$r = \frac{7.575}{3.94 \times 2.8}$

$= .67 \pm .0$

This plate shows method by which all relationship were computed. Regression in each of 34 Tables is linear and product-moment method is valid. TABLE K-2.